AIAA Guidance, Navigation, and Control Conference and Exhibit
15 - 18 August 2005, San Francisco, California

AIAA 2005-5995

Prospects and Challenges for Lorentz-Augmented Orbits

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A charged particle moving relative to a magnetic field accelerates in a direction perpendicular to its velocity and the magnetic field due to the Lorentz force. Although a negligible force for spacecraft potentials due to natural charging, this effect can be seen, for example, in the unusual celestial mechanics of dust particles in the rings of Saturn and Jupiter. This study evaluates the use of the Lorentz force as a means of orbit control for finite bodies, including small spacecraft. Although the Hamiltonian is constant in a frame that rotates with the earth, the co-rotational electrical field (associated with the rotating geomagnetic field) can do work, increasing or decreasing the orbit's semimajor axis. A number of intriguing applications are offered, including earth escape, drag compensation, new stable satellite formations, inclination control, nodal precession control, new sun-synchronous orbits, and non-Keplerian orbits for polar satellites. A candidate spacecraft design is offered based on lessons learned from decades of research in spacecraft charging and plasma interactions. The performance of this design is demonstrated by simulation. Closed-form equations for actuator sizing are provided.

Nomenclature

a	=	orbit semimajor axis
α	=	time-constant parameter from first-order ODE model of ionospheric charging
A	=	satellite cross-sectional (frontal) area
$\boldsymbol{B}$	=	magnetic induction (e.g. of the geomagnetic field)
B	=	magnitude of $\boldsymbol{B}$
B_0	=	magnetic induction at $r=r_0$
C	=	capacitance
C_d	=	coefficient of drag
$\boldsymbol{dF}$	=	perturbing force
δ	=	separation distance between two concentric spheres
E	=	satellite energy per unit mass
e	=	electron charge
ε_0	=	permittivity of free space
F_e	=	force on a single electron
$\boldsymbol{F}_L$	=	Lorentz force
F_r	=	perturbing force along the orbit's radial basis vector
F_t	=	perturbing force along the orbit's tangential basis vector
Φ_{gr}	=	gravitational potential function
H	=	Hamiltonian, the total mechanical energy in the rotating frame
h	=	magnitude of $\boldsymbol{h}$
$\boldsymbol{h}$	=	orbital angular momentum
I_{beam}	=	beam current
K	=	trailing-edge (TE) nondimensional angular deflection rate
λ_{De}	=	Debye length
m	=	spacecraft mass
μ	=	planetary gravitational constant
n_{e0}	=	quasi-neutral plasma concentration for electrons and ions
$\boldsymbol{p}$	=	generalized momentum

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P	=	pressure
q	=	charge
q_0	=	floating potential at the orbital altitude of interest
r	=	magnitude of $\boldsymbol{r}$
R	=	radius of a spherical conductor
$\boldsymbol{r}$	=	vector position of the spacecraft mass center with respect to the system barycenter
$\hat{\boldsymbol{r}}$	=	orbit's radial basis vector
r_0	=	mean planetary radius
σ	=	stress
σ_{yield}	=	yield stress
t	=	thickness
T_e	=	temperature of electrons with the Boltzmann distribution
T_i	=	temperature of ions with the Boltzmann distribution
$\hat{\boldsymbol{t}}$	=	orbit's tangential basis vector
V	=	electric potential
$\boldsymbol{v}$	=	velocity of a spacecraft in an planet-fixed frame
θ	=	coordinate of a spherical coordinate system (r, θ, ϕ) with origin at the planet's center and associated with a planet-fixed basis
$\boldsymbol{\omega}_e$	=	angular velocity of a planet-fixed frame with respect to an inertial frame
$\frac{^N d}{dt}$	=	operator indicating a vector derivative with respect to a frame N

I. Introduction

A particle that carries an electrical charge q with a velocity $\boldsymbol{v}$ relative to a magnetic field $\boldsymbol{B}$ experiences the Lorentz force $\boldsymbol{F}_L$:

$$\boldsymbol{F}_L = q\boldsymbol{v} \times \boldsymbol{B} \tag{1}$$

A consequence of relativistic electrodynamics, this force is used as a means of steering the electron beam in a cathode-ray tube; it is responsible for the orbital behavior of charged particles in a synchrotron; and, confirmed by data gathered at Jupiter and Saturn during the past two decades, has been shown to govern the orbital dynamics of dust in planetary rings.

In this paper we evaluate a new concept for propellantless spacecraft propulsion based on the Lorentz force. For this concept to be practical the spacecraft must be in the presence of a magnetic field, traveling at a significant relative velocity, and must be able to maintain a net electrical charge. The first and second requirements can be met in non-geostationary Earth orbit, at least within the magnetosphere, and near certain other celestial bodies (notably Jupiter, whose magnetic field strength is roughly 20,000 times that of the Earth). The issues that arise here involve primarily celestial mechanics. It turns out that a Lorentz-Augmented Orbit (LAO) may be used to address a number of relevant problems, including design of stable formations, orbit raising, inclination control, and drag mitigation. Meeting requirements associated with charging demands, for one, that careful attention be paid to electron and ion currents to the spacecraft in the presence of space plasma. It also demands that the risk of damage to spacecraft components by electrostatic discharge among differentially charged components be considered. This paper describes a candidate spacecraft architecture that holds the promise of meeting all of these requirements.

Before considering the details of what an LAO is, let us be clear about what it is not. Lorentz-force propulsion does not involve the use of electrodynamic tethers, although the physics is related. In the case of a tether, the current flowing through a long conductor induces a magnetic field that interacts with the planet's magnetic field to produce a force on the tether. Electrons travel through the tether at low speed but at as high a current as possible. For an LAO, the spacecraft may be very compact. The charge it carries travels at perhaps thousands of meters per second relative to the magnetic field, and this moving charge represents the current that results in a force similar to what a tether experiences. Furthermore, an LAO does not work though electrostatic levitation. That is, an LAO does not depend on Coulomb forces that act on the charged spacecraft. The use of intersatellite Coulomb forces in formation flying has been proposed[1,2], but in contrast an LAO requires no neighboring charged bodies. Formations of LAO-capable spacecraft are not designed to interact through Coulomb attraction or repulsion.

Decades of research in disparate areas of physics and engineering have bearing on the LAO concept. In addition to spacecraft dynamics and control, this research includes dusty plasmas (for example, in planetary ring formations), spacecraft charging, ion- and electron-beam experiments (conducted primarily for missile defense), high-voltage energy storage, and near-earth magnetospheric and plasma research. We provide an overview of some of the relevant work in these areas, although even a moderate degree of completeness is beyond the scope of this paper. Instead, we focus on LAO applications and try to justify the technical soundness of the concept by synthesizing results from these wide-ranging fields.

Despite a lengthy review of the literature in many fields, we find only one treatment of LAO orbit mechanics. It is in a study by M. E. Hough that addresses the impact of the Lorentz force on the comparatively short, ballistic trajectory of a missile, with no proposal of modulating the charge for the sake of control[3]. Our approach for satellites includes both control and the assumption that these forces may require a long time to effect a significant orbit change. So, despite the innovativeness of Hough's work, we look elsewhere for insight.

The most in-depth treatment of LAO celestial mechanics is found in the work of Schaffer, Burns, et al. on planetary dusty plasmas[4,5]. Their work offers explanations for gaps in Jupiter's and Saturn's rings that are based on identifying resonances in the orbit dynamics. The resonances arise thanks to interactions among gravity and small effects such as solar pressure, lunar perturbations, and the Lorentz force. The success of these studies validates models of particle charging and demonstrates that the Lorentz force leads to non-Keplerian orbits, at least for micron-size particles at a few Volts of potential. Dusty plasma researchers have also considered the problem of dust grains in orbit, but with a primary interest in plasmadynamics rather than celestial mechanics[6].

The literature on spacecraft charging is extensive. Much of it is concerned with the deleterious effects of differential charge, a problem that arises when dissimilar or discontinuous materials acquire potential as the spacecraft travels through the space plasma[7]. The photoelectric effect, in which photons cause some materials to emit electrons, can also be responsible. These potentials can result in arcing from one component to another or sputtering of surfaces and are to be avoided, whether by the choice of materials, charge management (by grounding to the surrounding plasma), or by careful placement of components and incorporation of Faraday cages[8,9]. These studies are concerned primarily with predicting charge levels and assessing the risks of material damage due to interactions with the natural environment. By contrast, we consider the active application of charge to a spacecraft body; and although the influence of the plasma is very relevant for the design LAO capable spacecraft, the equilibrium potentials achieved passively are not of direct interest here.

Another class of studies on spacecraft charging stems from ballistic missile defense research during the past two decades, much of it for the Strategic Defense Initiative of the 1980s. This work has involved high-energy ion and electron beam emission from sounding rockets and spacecraft. Some examples of rocket platforms include the Beam Experiment Aboard Rocket (BEAR), Space Power Experiment Aboard Rockets (SPEAR) I through III[10,11], and MAIMIK[12]. Satellite-based experiments include the high-altitude Application Technology Satellite 6 (ATS-6) and Spacecraft Charging at High Altitude (SCATHA) satellites. Theoretical studies have also been undertaken[13,14]. Some of the results of these studies are used in the analysis below. The Space Shuttle has also been the subject of charging studies, for example during the 1992 flight of the Tethered Satellite System (TSS)[15].

II. Equations of Motion

Augmented with the Lorentz Force, Newton's law of gravitation for a particle of mass m moving in the r^{-2} gravitational field of a point mass M becomes

$$m\frac{{}^N d^2}{dt^2}\boldsymbol{r} = -m\frac{\mu}{r^2}\hat{\boldsymbol{r}} + q\left(\frac{{}^N d}{dt}\boldsymbol{r} - \boldsymbol{\omega}_e \times \boldsymbol{r}\right)\times\boldsymbol{B}, \tag{2}$$

where the superscript N indicates a derivative taken with respect to a Newtonian, or inertial, frame, $\boldsymbol{r}$ is the vector position (magnitude r and direction $\hat{\boldsymbol{r}}$) of the particle relative to the system barycenter, $\mu = MG$ where G is the universal gravitational constant, q is the electric charge on the particle, $\boldsymbol{\omega}_e$ is the earth's angular velocity vector (herafter taken to be constant in N), and $\boldsymbol{B}$ is the magnetic field vector. This expression acknowledges that it is the particle's velocity relative to the magnetic field $\boldsymbol{v} = \frac{{}^N d}{dt}\boldsymbol{r} - \boldsymbol{\omega}_e \times \boldsymbol{r}$ that determines the Lorentz force. In the simplest model, the earth's magnetic field rotates with the earth. By relativity, this time-varying magnetic field represents an electric field, which is the means by which work can be done on the LAO.

In a frame E that rotates with the earth, the equation of motion in terms of the relative velocity $\boldsymbol{v} = \frac{{}^E d}{dt}\boldsymbol{r}$ and a gravitational potential Φ_{gr} is

$$\frac{{}^E d}{dt}\boldsymbol{v} = -\nabla\Phi_{gr} + \frac{q}{m}\boldsymbol{v}\times\boldsymbol{B} - 2\boldsymbol{\omega}_e \times \boldsymbol{v} - 2\boldsymbol{\omega}_e \times \left(\boldsymbol{\omega}_e \times \boldsymbol{r}\right), \tag{3}$$

where dividing through by *m* introduces the commonly used charge per mass $\frac{q}{m}$ as a parameter that determines the scale of the Lorentz force. Following Schaffer and Burns[16], we project this equation onto $\boldsymbol{v}$

$$\frac{{}^E d}{dt}\boldsymbol{v}\cdot\boldsymbol{v} = \frac{1}{2}\frac{d}{dt}v^2 = -\nabla\Phi_{gr}\cdot\frac{{}^E d}{dt}\boldsymbol{r} + \frac{d}{dt}\left(\frac{1}{2}r^2\omega_e^2\sin^2\theta\right), \tag{4}$$

where θ is a coordinate of a spherical coordinate system (r, θ, ϕ) with origin at the planet's center and associated with an E-fixed basis. Integrating between arbitrary t_1 and t_2 shows that the total mechanical energy in the rotating frame H is constant:

$$H(t_2) - H(t_1) = 0, \tag{5}$$

Schaffer and Burns point out that this function is the appropriate Hamiltonian in the noncanonical variables ($\boldsymbol{r}$, $\boldsymbol{p} = m\dot{\boldsymbol{r}}$) as demonstrated by Littlejohn[17,18], who used these coordinates in a perturbation theory for highly charged particles in slowly varying electromagnetic fields.

The existence of this constant Hamiltonian in a rotating frame suggests that only the rotation of the magnetic field, which causes the co-rotational electrical field, can do work in the inertial frame. By way of illustration, we consider the osculating elements of a restricted two-body orbit whose angular momentum is aligned with the planet's magnetic moment (i.e., a magnetic-equatorial orbit). The energy E per unit satellite mass in the inertial frame is given by

$$E = -\frac{\mu}{2a}, \tag{6}$$

where a is the orbit's semimajor axis. Its time rate of change is

$$\dot{E} = \dot{a}\frac{\mu}{2a^2} = \frac{{}^N d}{dt}\boldsymbol{r}\cdot\boldsymbol{dF}, \tag{7}$$

a familiar result in which the energy depends entirely on the semimajor axis. Work is done if and only if the semimajor axis changes. The perturbing force $\boldsymbol{dF}$ per unit mass is

$$\boldsymbol{dF} = F_r\hat{\boldsymbol{r}} + F_t\hat{\boldsymbol{t}}. \tag{8}$$

in a basis where $\hat{\boldsymbol{t}}$ indicates a tangential force (normal to $\boldsymbol{r}$ and the orbital angular momentum). In this basis,

$$\frac{{}^N d}{dt}\boldsymbol{r} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\phi}\hat{\boldsymbol{t}}, \tag{9}$$

where ϕ is the true anomaly. Therefore,

$$\dot{a}\frac{\mu}{2a^2} = \left(\dot{r}\hat{\boldsymbol{r}} + r\dot{\phi}\hat{\boldsymbol{t}}\right)\cdot\left(F_r\hat{\boldsymbol{r}} + F_t\hat{\boldsymbol{t}}\right), \tag{10}$$

or

$$\dot{a}\frac{\mu}{2a^2} = \dot{r}F_r + r\dot{\phi}F_t \cdot \tag{11}$$

At the magnetic equator $\boldsymbol{B} = B\hat{\boldsymbol{r}}\times\hat{\boldsymbol{t}}$. In this case, the radial force is

$$F_r = \hat{\boldsymbol{r}}\cdot\left[\frac{q}{m}\left(\frac{{}^N d}{dt}\boldsymbol{r} - \boldsymbol{\omega}_e\times\boldsymbol{r}\right)\times\boldsymbol{B}\right] = \frac{qB}{m}r\left(\dot{\phi} - \omega_e\right), \tag{12}$$

And the tangential force is

$$F_t = \hat{\boldsymbol{t}}\cdot\left[\frac{q}{m}\left(\frac{{}^N d}{dt}\boldsymbol{r} - \boldsymbol{\omega}_e\times\boldsymbol{r}\right)\times\boldsymbol{B}\right] = -\frac{qB}{m}\dot{r}\cdot \tag{13}$$

The semimajor axis therefore changes according to

$$\dot{a} = -\frac{qB}{m}\frac{2a^2}{\mu}r\omega_e \cdot \tag{14}$$

The exclusive dependence on the earth spin rate ω_e confirms that it is only the co-rotational effect that does work. The signs of *q* and *B* are important here: positive charge (due to, say electron emission) and negative (southward) magnetic fields cause a loss in orbital energy, as does negative charge in a positive (northward) magnetic field. Only when *q* and *B* are of opposite sign is the energy change positive. Although this result would indicate that greater Lorentz force is available at higher altitude, in fact *B* drops off approximately with r^3; so the net effect of increased altitude is deleterious.

The Lorentz Force cannot exist in several other special cases. One is the case where

$$\frac{{}^N d}{dt}\boldsymbol{r} - \boldsymbol{\omega}_e\times\boldsymbol{r} \parallel \boldsymbol{B}, \tag{15}$$

a situation that arises when a polar satellite crosses the equator. Another is

$$\frac{{}^N d}{dt}\boldsymbol{r} = \boldsymbol{\omega}_e\times\boldsymbol{r}\cdot \tag{16}$$

These kinematics describe an orbit in which the satellite is stationary in the rotating E frame. In the case of the Earth with an assumed dipolar magnetic field, this orbit is at geostationary altitude. But for the tilt of the geomagnetic dipole (roughly 10.7^o from true north), no force is available at GEO. Nevertheless, at this altitude and inclination, an LAO may still experience an energy change if the orbit is retrograde. In fact, for a given initial orbital energy, retrograde motion may be a more powerful means of providing earth escape (as discussed below).

III. Applications

A propellant-based system for orbit control can deliver only finite ΔV. By contrast, devices like electrodynamic tethers, solar sails, and LAO capable spacecraft offer the prospect of indefinite thrust, all at some cost (such as technical risk, operational schedule, and development budget). Here we evaluate some LAO applications that, we hope, illustrate its value as a low-thrust propulsion concept. The next section addresses implementation issues that arise in these applications.

A. Drag Compensation

Aerodynamic drag acts on a satellite in a direction opposite its velocity. Since the Lorentz force is always perpendicular to this direction, one might be tempted to conclude that the Lorentz force cannot be used to cancel the effects of drag. Specifically, drag reduces the angular momentum $\boldsymbol{h}$ of the orbit according to

$$\frac{{}^N d}{dt}\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{F}_d \,, \tag{17}$$

or in the orbit-normal direction,

$$\dot{h} = rF_d \,. \tag{18}$$

This force reduces both the orbital energy and the angular momentum.

The Lorentz force includes a tangential component whenever

$$\hat{\boldsymbol{r}} \cdot \frac{{}^N d}{dt}\boldsymbol{r} \neq 0 \,, \tag{19}$$

or equivalently $\dot{r} \neq 0$, i.e. in an elliptical orbit. So, drag compensation by the Lorentz force in circular orbits is impossible. However, in an elliptical orbit both energy and momentum may be changed at some point in the orbit, and drag compensation may be possible. This possibility is illustrated with an example. For this cursory look, we consider a drag force

$$F_d = \left(\frac{1}{2}\rho C_d A\right)\frac{{}^N d}{dt}\boldsymbol{r} \cdot \frac{{}^N d}{dt}\boldsymbol{r} \,, \tag{20}$$

where ρ is the atmospheric density at $\boldsymbol{r}$, Cd is the coefficient of drag (about 2.2 for common satellites[19]), and A is the frontal cross-sectional area of the spacecraft.

As discussed in section IV, a candidate LAO capable spacecraft is one in which core subsystems may be surrounded by a thin spherical conducting shell. A 1.5m radius sphere has cross-sectional area A=7.07m^2. Based on density from the U.S. Standard Atmosphere[20], Figure 1 shows the change in orbital angular-momentum magnitude and orbital energy for an LAO capable spacecraft with perigee height 250 km and varying eccentricity. We assume q/m=1 C/kg, which may be difficult to realize in practice but illustrates the trend well. We further assume that both positive and negative potentials can be achieved because the charge must switch sign at perigee and apogee for optimal efficiency. The magnetic field is taken as a dipole, with

$$\boldsymbol{B}(r) = \frac{B_0 r_0^3}{r^3}\left(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{t}}\right) \approx \frac{8.04 \times 10^{15}}{r^3}\left(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{t}}\right), \tag{21}$$

The figure shows that the Lorentz force is effective at making up for lost angular momentum, which is possible for nearly circular orbits. It is less effective in making up for lost energy, which becomes feasible for more highly elliptical orbits (e>0.18). Choosing when to establish the charge, and at what polarity, allows one to balance energy and momentum so that the desired eccentricity and semimajor axis may be maintained. While the effect is not large, the benefits are most pronounced for geostationary transfer orbits or supersynchronous transfer orbits, where q/m=0.1 C/kg and lower can make up for momentum and energy losses due to drag at perigee passage.

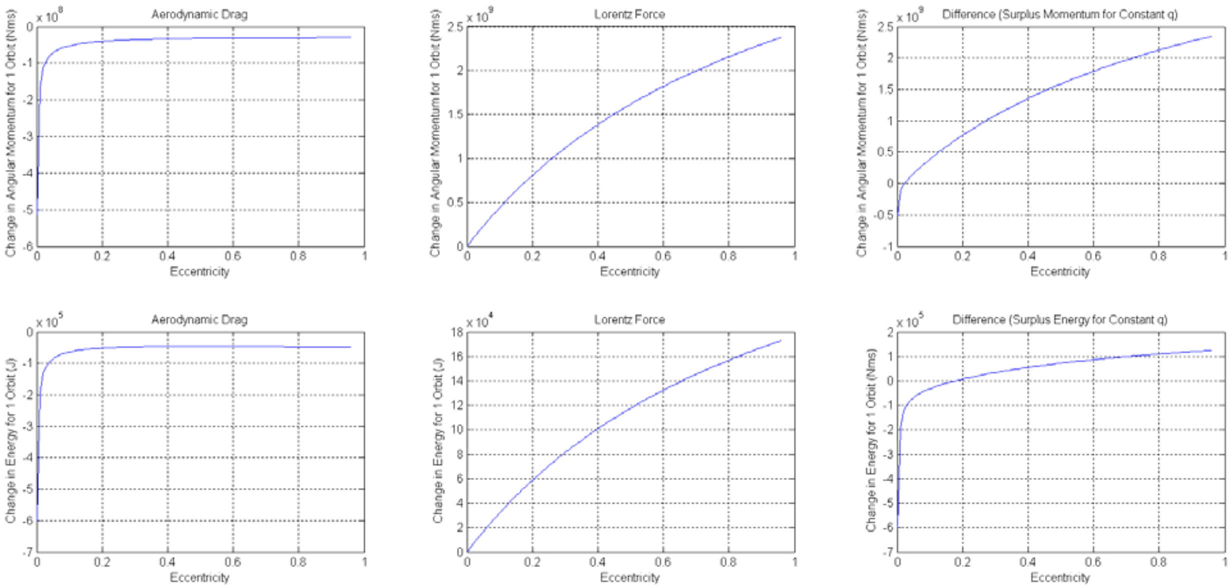


Figure 1. Lorentz Force Drag Compensation: Angular Momentum and Energy Correction with *q/m*=1 C/kg at Magnetic Equator; Osculating Elements with Perigee Radius=250 km.

Half this performance is possible if only one polarity can be established. The reason is that true anomalies past π, where the kinematics are a mirror image, require an opposite charge to achieve the same effect.

Although this example depended on a specific cross-sectional area, the spacecraft design described below can hold charge in proportion to its cross-sectional area. Therefore, the ratio of drag-force effects to Lorentz-force effects is constant for a given *q/m*. This proof-of-concept analysis shows that Lorentz-based drag compensation can correct for a significant portion of the loss in energy and momentum due to drag over the course of an orbit. The compensation is not instantaneous but can be complete by the time half an orbit has passed.

Because the LAO's energy is not constant, the Lorentz force may be used to achieve earth escape. With *q/m*=1 and neglecting drag (not a good assumption, but helpful for illustrative purposes), earth escape from Geostationary Transfer Orbit requires only a little over a year. The evolution of the orbit is suggested in Figure 2.

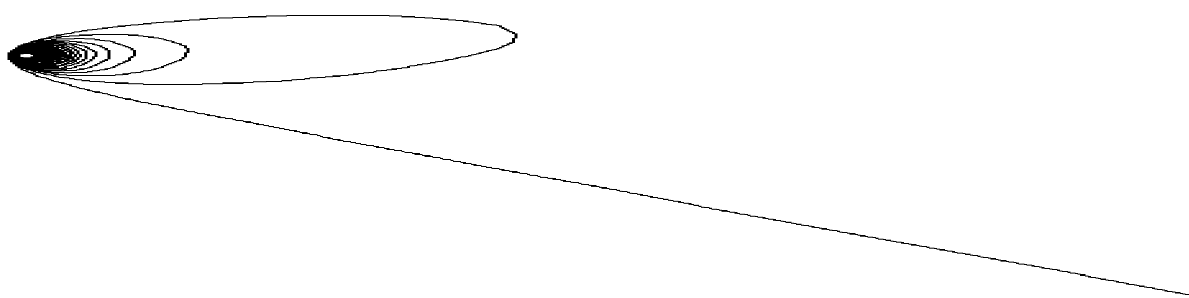


Figure 2. Orbit Evolution for Earth Escape from GTO; *q/m*=1 C/kg.

B. Satellite Formations

Satellite formations offer the prospect of large-scale space-based interferometry and sparse-aperture telescopes. Programs such as LISA[21] and TPF[22] may depend on precise formations and, of course, benefit from fuel-efficient designs. An LAO can be used to create such formations, at least within a magnetic field. The radial force applied to a satellite in a circular LAO alters the spacecraft's potential energy—not by much, but enough to establish a formation of spacecraft with different altitudes and identical orbit angular rate ω. Using the earlier inverse-cube model for $\boldsymbol{B}$ we find that circular orbits of a given period can be achieved at radii

$$r = \sqrt[3]{\frac{1}{\omega^2}\left(\mu - \frac{q}{m}(\omega - \omega_e)B_0 r_0^3\right)}. \tag{22}$$

Figure 2 shows the radial distance between two satellites in equatorial circular orbits, one with 0.001 C/kg charge, and one with no electrical potential. The curve goes through 0 at geostationary altitude because, as explained above, at that height the spacecraft exhibits no velocity relative to the rotating magnetic field (and therefore experiences no Lorentz force). Absent perturbations, this formation is in equilibrium and requires no propellant for stationkeeping.

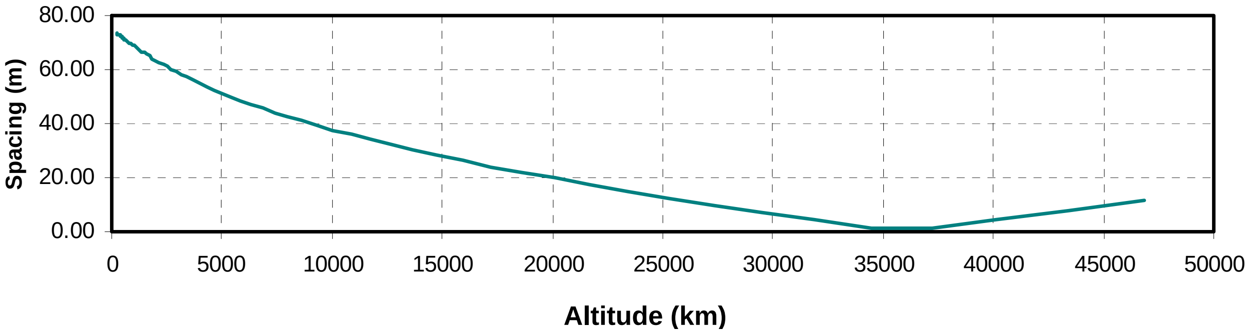


Figure 3. Vertical Spacing for Circular Prograde Orbits: q/m=0.001

Numerous formations can be achieved with such a capability: for example, a string-of-pearls style constellation in which the satellites form a straight line across the circular orbit. Rotating formations (based on modified Clohessy-Wiltshire equations) may be able to form a paraboloid arrangement of spacecraft for sparse optical observation of terrestrial or astronomical targets. Research in this area is on-going.

C. Rendezvous

A related phenomenon is that spacecraft with different potential can co-exist in a circular orbit but travel at different velocity. There are two solutions for the velocity (depending on the polarity of the charge and the choice of prograde vs. retrograde motion):

$$\omega = -\frac{q}{m} B_0 \frac{r_0^3}{r^3} \pm \frac{\sqrt{\left(\frac{q}{m} B_0 r_0^3\right)^2 + 4r^3\left(\mu + \frac{q}{m}\omega_e B_0 r_0^3\right)}}{2r^3}. \tag{23}$$

Figure 4 shows the maximum time for one satellite with q/m=0.001 C/kg charge to catch up with another in the same circular orbit (both spacecraft prograde or both retrograde), using only the Lorentz force as propulsion. This worst case represents a situation in which one satellite begins 180^{o} in true anomaly from the other. This application is better suited for low-earth orbiting satellites, but with higher charge per mass, even MEO satellites might be serviced by a spacecraft in this fashion.

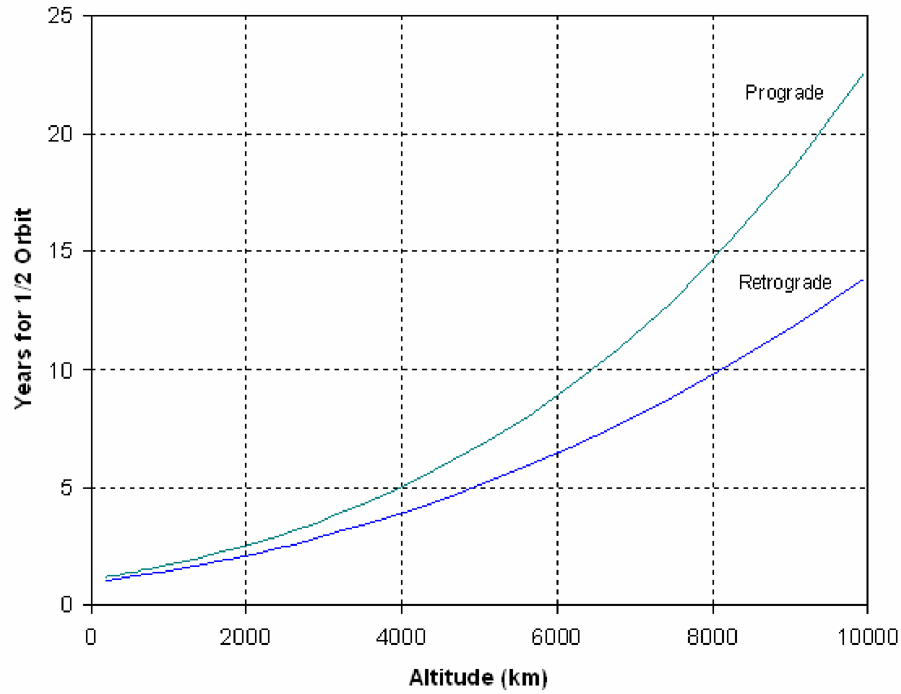


Figure 4. Time to Rendezvous (years) in Circular Orbit: q/m=0.001

D. Inclination Control

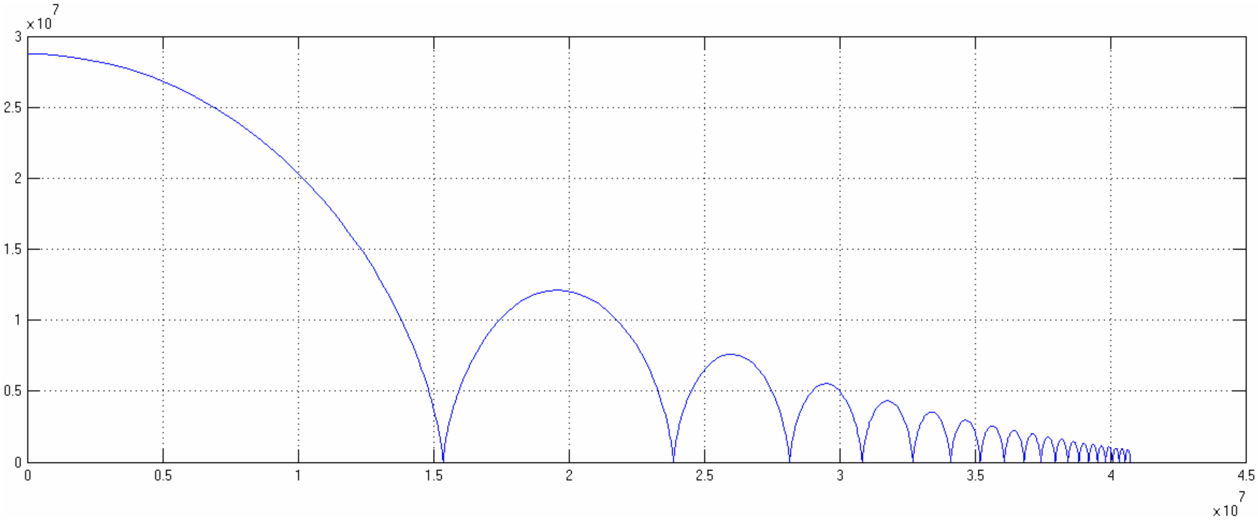
Changing inclination is expensive for spacecraft that must expel mass for propulsion. Inclination change with the Lorentz-force is equally difficult—taking considerable time because forces are small—but it requires no expendables. The Lorentz force can be used to change inclination anywhere that the magnetic field is not parallel to the orbital angular momentum, with the additional constraint that the velocity in the E frame must also not be parallel to the magnetic field. I.e. inclination change requires $\boldsymbol{B} \times \boldsymbol{h} \neq 0$ and $\boldsymbol{B} \times (\boldsymbol{v} - \boldsymbol{\omega}_e \times \boldsymbol{r}) \neq 0$. An equatorial orbit violates the first condition; one can infer that inclination is in a stable equilibrium at the magnetic equator. In a polar orbit $\boldsymbol{B} \parallel \boldsymbol{v}$ at the equator, and the only effect that can influence inclination is $\boldsymbol{\omega}_e \times \boldsymbol{r}$. However, at the equator $\boldsymbol{\omega}_e \times \boldsymbol{r}$ is nearly perpendicular to ***B***; so we conclude that not much useful torque can be applied there. Elsewhere in a polar orbit the Lorentz force can apply a moment. In fact, at the poles, the moment is strongest both because ***B*** is concentrated there and because $\boldsymbol{r} \times (q\boldsymbol{v} \times \boldsymbol{B}) \perp \boldsymbol{h}$.

E. Jupiter Missions: JIMO capture and Orbit Insertion Drag Compensation

Other applications associated with non-equatorial orbits include nodal precession, new sun-synchronous orbits, and non-Keplerian (i.e. non-planar) polar orbits. Interplanetary missions, using the sun's magnetic field are also a possibility. Rather than explore these applications in detail here, we offer one final concept: capture of the JIMO spacecraft at Jupiter.

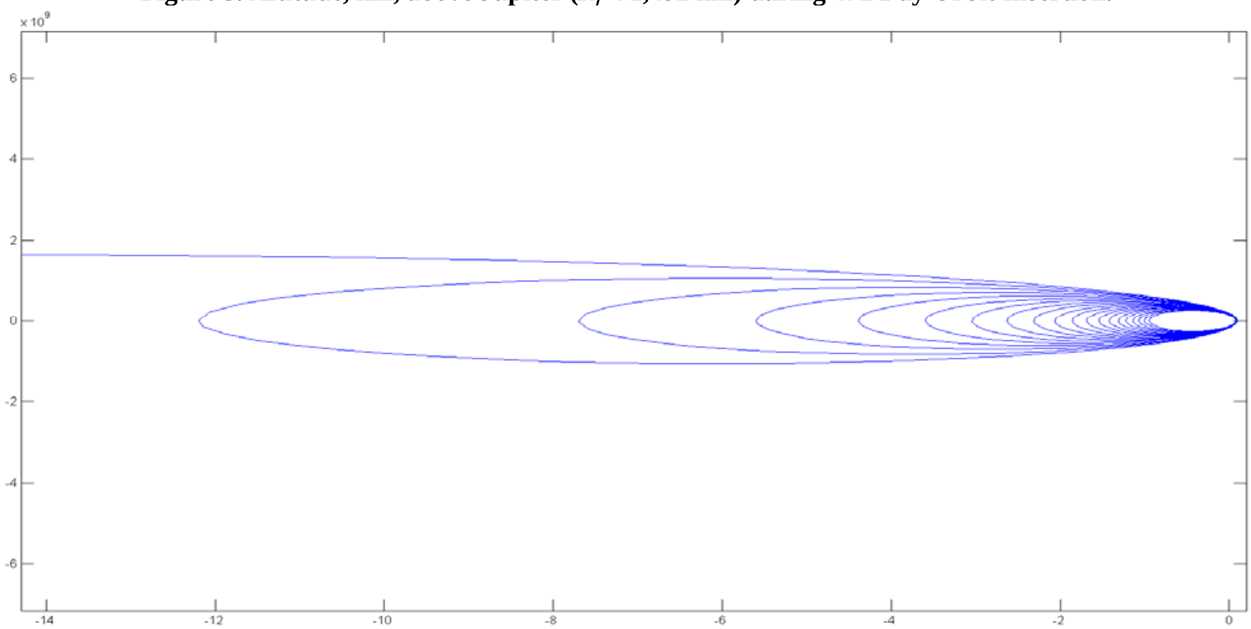
Jupiter's magnetosphere is roughly 20,000 times stronger than earth's. This high flux density suggests that the Lorentz force can be very effective for orbiting spacecraft. Furthermore, Jupiter's rotation rate is higher than earth's (1.76×10^{-5} rad/sec, for a period of about 9.93 hours). This higher rate makes the co-rotational electric field more powerful, offering the prospect of faster energy change near the planet. The electron and ion densities near the planet are also correspondingly high[23]. Charging due to natural interactions with this plasma may offer the prospect of little or no hardware dedicated to charging the spacecraft at Jupiter. These benefits prompt one to consider the prospect of using Lorentz-force braking to capture a spacecraft bound for Jupiter's orbit. The Jupiter Icy Moons Orbiter (JIMO) is a candidate mission.

In this example we assume q/m=0.01 C/kg. JIMO is initially in a parabolic orbit. Its first perijove passage is at an altitude of 20,000 km. After 472 days, perijove is 1000 km, and apojove is 944,000 km, about 14 Jupiter radii, within the orbit of Ganymede. Figure 5 shows the altitude, and Figure 6 shows the shape of the orbit.



Time, seconds

Figure 5. Altitude, km, above Jupiter (R_j=71,492 km) during 472 Day Orbit Insertion.



Distance, 10^9 m

Figure 6. Orbit Evolution During Jupiter Insertion. Scale in 10^9 m.

IV. Space System Design

The objective of designing an LAO spacecraft is to provide maximum acceleration for limited electrical power. Because the Lorentz force is proportional to the charge, maximizing q/m is a natural goal. However, the rate at which this charge is lost to the environment determines the power required to keep the spacecraft charged, and the susceptibility of a design to discharge is therefore also of interest.

Spacecraft naturally acquire charge in earth orbit. The ambient plasma is partly responsible: ion and electron currents to the spacecraft can create high enough potentials to damage materials and electrical components. The photoelectric effect, which causes materials to emit electrons, also plays a role. In earth eclipse, where the photoelectric effect is virtually nonexistent, spacecraft tend to charge negative because of collisions with free electrons. The plasma is far denser at low-earth orbits, where (regrettably) the Lorentz force can be strongest for a given q/m because the magnetic field is also strongest there.

It is important to distinguish between the concept of a body with potential V relative to the plasma and the concept of charge as applied to a capacitor. The capacitance C of an LAO spacecraft is clearly of relevance: the charge q it carries goes with $q=CV$. However, the capacitance one commonly encounters in electrical engineering is the result of energy storage via two oppositely-charged surfaces. A capacitor of this type, one which contains both positive an negative charge, does not experience a net Lorentz force (although individual components of it may). Very high-capacitance, low-mass devices like Maxwell Technologies' Boostcap Ultracapacitors offer over 2000 C at 2.5V for 0.5 kg[24], but these devices offer no net electrical field because the charges inside them cancel.

Instead, the charge on an LAO spacecraft is likely best achieved on the surface of a conducting sphere. The sphere's shape precludes charge concentrations, which would otherwise encourage arcing and discharge into the plasma. Perhaps most important, surrounding the spacecraft with a conducting sphere establishes a Faraday cage that completely protects components in the interior from electrical discharge (because there can be no electrical field inside a conducting surface[25]). This sphere has the added benefit that it may be deployed by inflation, as have other metallized space inflatables[26], for a lightweight solution that is compact for launch. Practical considerations (such as communications) will require a hole in this shield, but small-enough holes do not affect the Faraday cage's ability to shield electric fields.

Figure 7. 1.75 m Radius Optical Calibration Sphere from L'Garde Inc., Launched January 6, 2000.

The time constant for spacecraft charging is surprisingly low. In less than a second a spacecraft may reach a "floating" potential, which is maintained by interactions with the environment. This equivalent RC circuit turns out to be the most important sizing parameter for LAO capable spacecraft. Consistent previous work in the area of spacecraft charging, we model the dynamics of charging as a first-order system:

$$\dot{q}(t) = -\alpha(q - q_0) + I_{beam}, \tag{24}$$

where a is a constant that depends on the plasma environment, q_0 is the floating potential, and I_{beam} is the current from an electron or ion beam used to counteract the effects of plasma charging.

Data from the SPEAR 1 experiment helps establish α at low altitudes (200-350 km)[27]. Figure 7 shows the charge decay of a rocket body with V=42 kV. Approximated as a linear decay, and with I_{beam}=0, this data suggests a=1.35s^{-1}. Rustan et al, Figures 7 and 8, show the current vs. voltage during experiments conducted on this flight. The high voltage requires only 0.06 A, or 2520 W, if one assumes that the power is associated with a beam of this energy (42 keV).

Debye shielding is a phenomenon that occurs for charged bodies in a plasma. Oppositely charged particles are drawn to the vicinity of the charged body but do not adhere because of their high thermal energy. The Debye sheath works in the favor of an LAO spacecraft, increasing its capacitance. The reason is that the capacitance of a sphere in a vacuum is

$$C = 4\pi\varepsilon_0 R\,, \tag{25}$$

where ε_0=8.8542×10^{-12} is the permittivity of free space, and R is the radius of the capacitive sphere. For two concentric spheres,

$$C = 4\pi\varepsilon_0 \frac{R(R+\delta)}{\delta}\,, \tag{26}$$

where δ is the separation between them. In the case of the sphere in a plasma, the separation is taken to be a few Debye lengths λ_{De}, which in low-earth orbit is about 1 cm:

$$\lambda_{De} = \sqrt{\frac{\varepsilon_0}{n_{e0}e^2\left(1/T_e + 1/T_i\right)}}\,, \tag{27}$$

where e is the electron charge 1.602×10^{-19} C, n_{e0} is the quasi-neutral plasma concentration for electrons and ions as explained in Fridman and Kennedy, and one assumes the Boltzmann distribution for electrons (temperature T_e) and ions (temperature T_i)[28].

This phenomenon increases the capacitance of the LAO spacecraft over that of a sphere in a vacuum by the ratio $\frac{(R+\delta)}{\delta}$, which can be a large value. In the example of a 1.5 m sphere in LEO, the increase is a factor of 151. The beneficial effect of the Debye sheath diminishes as one considers higher-altitude satellites. At GEO altitudes, for example, λ_{De} may be many meters, and its contribution to the capacitance is negligible for spheres of manageable size. The particles in the Debye sheath do not travel with the charged body in the way one might expect. If they did, their opposite charge would cancel the Lorentz force on the charged body, resulting in no net acceleration. As evidence that this cancellation does not occur, recall that the particles in Jupiter's rings have been shown to experience the Lorentz force; therefore, their Debye sheaths do not prevent them from accelerating. In fact, for a body traveling at high speed like the LAO spacecraft, the Debye sheath is elongated, like a Mach cone[29], and its precise effects on the capacitance are somewhat more subtle. We approximate this exact shape with a sphere under the assumption that the increased capacitance due to the compressed potential field in front of the satellite balances out the loss of capacitance that results from the attenuated field behind it. Nosenko et al. provide images of this phenomenon.

Establishing the charge on the surface of the sphere may be achieved in several ways. Interaction with the natural environment causes some charge to build up, but this phenomenon cannot be used for control. One reason is that the plasma density and temperature fluctuates wildly[30]. Another is that polarity changes are available only due to earth eclipse but, as evidenced by the examples, a successful LAO requires more versatility. The most likely means of active charge control is the use of electron or ion beams. Technologies for ion propulsion may be adapted here, but if that hardware is already available, and the design limitations imposed by finite propellant are acceptable, there would seem to be little reason to use ion expulsion for this less-capable technique. Electron-beam emission would seem to be the better choice. First, electrons need not be stored as propellant. Also, electrons are orders of magnitude lighter than any ion, and so the momentum imparted to the particles to achieve a certain beam current is significantly lower. They are also easily manipulated with small magnetic fields, as is the case in a cathode-ray tube. The choice of only a single charge species doubles the duration of maneuvers (assuming a symmetric orbit, i.e. the approximation of osculating two-body orbital elements remains in force), but control over the same elements is still possible. Discharging in the space environment is trivially easy, requiring only a few seconds without beam current, as suggested by the SPEAR experiments.

We propose storing the charge on the surface of a sphere, from which a partly insulated plasma contactor extends. Figure 8 is a sketch of the concept. The sphere is envisioned as an inflatable, say Kapton, to which is bonded a transparent conductive film. Transparency allows some sunlight to enter the sphere, offering the possibility of operating a solar array within the Faraday cage.

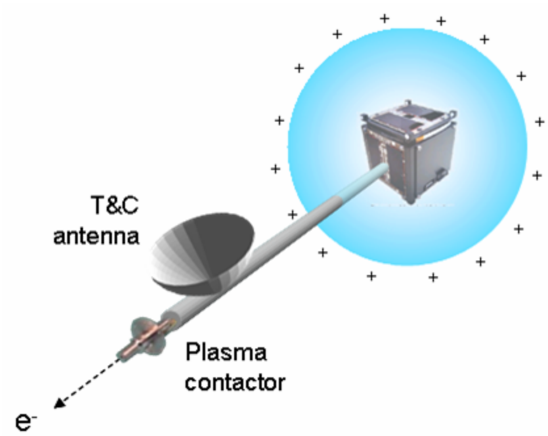


Figure 9. LAO Spacecraft Concept

The charge built up in this fashion may be deposited on the surface of the sphere directly, through simple harness, or through a belt mechanism like that of a Van De Graaff generator[31]. Small electrostatic generators of this familiar variety store megavolts of charge on the surface of a sphere in air for very low current. Figure 9 shows a small, lightweight generator capable of approximately 200 kV. Rather than depending on a heavy AC motor, the LAO spacecraft would likely use a plasma contactor with a high-voltage electrode.

Figure 9. 200 kV Van de Graaff Generator from Science First Inc.

Important considerations for sizing the sphere include several conflicting design goals. To minimize mass, the sphere must be as light as possible; however, lightweight, thin materials suffer from low tensile strength. Tensile strength is important here because the electrostatic charge applies a negative (radial outward) pressure to the exterior of the sphere. From the force on a single electron at the surface

$$F_e = \frac{eq}{4\pi\varepsilon_0 R^2}, \tag{28}$$

we find that the pressure is

$$P = \varepsilon_0 \left(\frac{(R + \lambda_{De})V}{R\lambda_{De}} \right)^2, \tag{29}$$

and based on the formula for stress σ in a thin-walled sphere of thickness t,

$$\sigma = \frac{PR}{2t}, \tag{30}$$

the stress in terms of relevant LAO parameters is

$$\sigma = \frac{\varepsilon_0 (R + \lambda_{De})^2 V^2}{2R\lambda_{De} t}. \tag{31}$$

The mass m of the spacecraft depends on the mass of this sphere ($4\pi R^2 \rho t$) given its material density ρ and on some unrelated components M_0. Combining these results leads to an expression for the LAO spacecraft's capability limit $\left(\frac{q}{m}\right)_{\max}$ in terms of the material yield strength σ_{yield}:

$$\left(\frac{q}{m}\right)_{\max} = \frac{4\pi\varepsilon_0 R (R + \lambda_{De}) V}{\lambda_{De}\left(M_0 + 4\pi R^2 \rho \left(\frac{\varepsilon_0 (R + \lambda_{De})^2 V^2}{2\sigma_{yield} R \lambda_{De}} \right)\right)}. \tag{32}$$

For example, a 3m sphere made of low-density (900 kg/m^3) high-tensile strength (2.8×10^{10} Pa at failure) Polypropylene Homopolymer (FF-026-M) from Sunoco Chemicals can hold a 1 MV charge for about 0.054 kg. The result is q/m=0.03 C/kg for a 1 kg cubesat. By contrast, a steel sphere would require 7.5 kg of material at the likely unrealizable thickness of 8.3 microns.

The final issue noted here concerns the shape of the geomagnetic field. Toward the sun, the magnetosphere is compressed by the solar wind. This bow shock and the magnetotail (on the other side of the earth) add complexity to the operations concept but may also be useful. Careful design of the operations concept for an LAO-capable spacecraft must take into account the limited choices for the line of apsides. While near-earth orbits can be designed conceptually for the dipole approximation, missions that depend on the Lorentz force at high altitudes must consider how to modulate the charge to accommodate, or take advantage of, the peculiarities of this shape.

V. Conclusions

Material limits constrain the maximum charge per mass that can be achieved. For the best material available, and with a sphere size of 3m radius, q/m=0.03 C/kg seems to be the maximum. For this limit, earth escape and drag compensation are out of the question. However, less-demanding scenarios, such as formations and Jupiter insertion, are well within material capabilities. In that case, less efficient but less exotic materials, like VDA Kapton, may even be used for the Faraday cage. If a more efficient charge-storage system than a conductive sphere can be developed, this constraint might be relaxed, and remarkable feats such as earth escape without propellant may be realized.

Acknowledgments

The author would like to thank Mr. Brett Streetman and Professor Joseph Burns, both of Cornell University, for valuable discussions on this subject.

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