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Technical Notes

Initial Perturbation Amplitude of Liquid Sheets Produced by Jet-Impingement Nozzles

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Nomenclature

d = diameter
f = breakup parameter
h = sheet thickness
K = sheet parameter
Oh = Ohnserge number
R = jet radius
Re = Reynolds number
r = distance from impingement point to plate tip
U = velocity
We = Weber number
η = wave amplitude
μ = viscosity
ρ = density
σ = surface tension

Subscripts

D = droplet
g = gas
j = jet
L = ligament
l = liquid
m = mean
p = plate

I. Introduction

THE liquid-film atomization nozzles have numerous applications, such as in gas turbines, liquid-fueled rocket engines, automobile engines, and recovery boilers in the pulp and paper industry. One method of film formation is by impinging a liquid jet on a solid surface. This method has several advantages, such as low injection-pressure loss and high controllability of the generated liquid film [1]. In this type of nozzle, a jet of fluid impinges on a splash plate and spreads out radially while thinning. The formed liquid sheet breaks into small droplets under the influence of the surrounding medium, turbulence, etc.

There have been numerous studies on the sheet breakup process [2–12], which have identified different modes of disintegrations, droplet size distribution, etc. The sheet may break due to aerodynamic wave instability, laminar edge instability, laminar tearing, turbulent edge instability, or turbulent tearing and perforation. The sheet instability is described based on an instability theory using both linear and nonlinear theories. The linear instability theory considers the growth of infinitesimal disturbances due to aerodynamic stresses on the surface of a liquid sheet due to disturbances originated in the pipe and at the injection time. The wave with the maximum growth rate is assumed to cause the breakup of the sheet. Various relations have been developed for the maximum growth rate of such disturbances. Essentially, all the proposed models provide a perturbation amplitude in the following form:

$$\eta = \eta_0 e^{\alpha_{\max} t} \tag{1}$$

where $\alpha_{\max}$ is the maximum growth rate of the instability wave. The growth rates obtained based on the linear theory are independent of the initial disturbance amplitude. However, to find the breakup time and breakup length, we need to use Eq. (1), which requires η_0, the initial disturbance amplitude, which is generally not available. To overcome this problem, the experimentally measured breakup time or breakup length are used to estimate a value for the initial disturbance amplitude, which in fact represents an empirical parameter adjusting the theoretical model to match the experimental values. This Note examines the validity of the commonly used parameters for the initial amplitude and provides a new procedure to determine the initial amplitude with improved accuracy and universality.

II. Initial Wave Amplitude

The breakup time of a liquid sheet can be predicted using the growth rate of a perturbation wave and its initial amplitude. Inversely, the breakup time can also be used to infer initial perturbation amplitude. The breakup time of a liquid sheet can be found based on the assumption that breakup occurs when the disturbance amplitude becomes the same as the sheet thickness, $\eta = h$. The result is

$$t = \frac{L}{U} = \frac{1}{\alpha_{\max}} \ln\left(\frac{h}{\eta_0}\right) = \frac{f}{\alpha_{\max}} \tag{2}$$

where f is referred to as the breakup parameter and it is determined using experimentally found breakup times. The liquid ligament and droplet sizes can then be determined using the wavelength responsible for the breakup. Dombrowski and Johns [12] assumed that the liquid sheet breaks up into cylindrical ligaments, which in turn become unstable and break into droplets. They provided the following equation for the ligament size produced by the instability and breakup of a two-dimensional attenuating liquid sheet:

$$d_L = 2\left(\frac{4}{3f}\right)^{1/3} \left(\frac{k^2\sigma^2}{\rho_g \rho_L U_s^2}\right)^{1/6} \left[1 + 2.6\mu_l \sqrt[3]{\left(\frac{k\rho_g^4 U_s^8}{6f\rho_l^2\sigma^5}\right)}\right]^{1/5} \tag{3}$$

where, in the case of jet-impingement nozzles, k is the thickness parameter at the plate tip, defined as

$$k = h_p r_p \tag{4}$$

Dombrowski and Johns [12] successfully applied their model to predict droplet size produced by a fan spray. Inamura et al. [1] used

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