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Globally Convergent Optimization Algorithm Using Conservative Convex Separable Diagonal Quadratic Approximations

Albert A. Groenwold* and Derren W. Wood†
University of Stellenbosch, Stellenbosch 7602, South Africa
and
L. F. P. Etman‡ and Simon Tosserams§
Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands

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We implement and test a globally convergent sequential approximate optimization algorithm based on (convexified) diagonal quadratic approximations. The algorithm resides in the class of globally convergent optimization methods based on conservative convex separable approximations developed by Svanberg. At the start of each outer iteration, the initial curvatures of the diagonal quadratic approximations are estimated using historic objective and/or constraint function value information, or by building the diagonal quadratic approximation to the reciprocal approximation at the current iterate. During inner iterations, these curvatures are increased if no feasible descent step can be made. Although this conditional enforcement of conservatism on the subproblems is a relaxation of the strict conservatism enforced by Svanberg, global convergence is still inherited from the conservative convex separable approximations framework developed by Svanberg. A numerical comparison with the globally convergent version of the method of moving asymptotes and the nonconservative variants of both our algorithm and method of moving asymptotes is made.

Nomenclature

c_2	=	curvature
f_j	=	constraint function j, $j > 0$
$\tilde{f}_j$	=	approximate constraint function j, $j > 0$
f_0	=	objective function
$\tilde{f}_0$	=	approximate objective function
k	=	outer iteration number
$\mathcal{L}$	=	Lagrangian
l	=	inner iteration number
m	=	number of constraints
n	=	number of design variables
P_D	=	dual subproblem
P_P	=	primal subproblem
$\mathcal{R}$	=	real number
$\boldsymbol{x} \in \mathcal{R}^n$	=	primal (design) variables
$\check{x}_i$	=	lower bound of design variable i
$\hat{x}_i$	=	upper bound of design variable i
α	=	index; runs over 0 to m
β	=	intermediate result
γ	=	dual
ϵ	=	tolerance
$\boldsymbol{\lambda} \in \mathcal{R}^m$	=	dual variables
χ	=	multiplication constant

I. Introduction

In structural optimization, sequential approximate optimization (SAO) methods are the optimization methods of choice, in particular when computationally demanding analysis models are part of the optimization loop. These methods typically use a tailored approximation with nonlinear curvature for not only the objective function, but each and every constraint. The specific SAO algorithm used is often selected purely as a result of experience with that algorithm for a specific class of programming problems, irrespective of the existence of proof of global convergence for that algorithm.

Well-known globally convergent algorithms from mathematical programming are sequential quadratic programming (SQP) algorithms. These methods are very efficient (where efficiency is expressed in terms of the number of function and gradient evaluations required for convergence and termination). The efficiency, however, derives from the use of second-order information, which is often unattractive in structural optimization, because second-order information requires the storage of the, in general, fully populated Hessian matrix. This is highly demanding of resources for the very large number of design variables often present in structural optimization.

To avoid the storage of coupling terms, most of the approximations popular in structural optimization are based on separable subproblems, for example, see the review paper by Barthelemy and Haftka [1]. In addition, several of these algorithms are also based on *convex* subproblems. (This is certainly the case for algorithms based on dual principles, because the combination of separability and convexity often allows for the formulation of highly efficient dual formulations.)

Proof of global convergence has never been demonstrated for many an implementation of popular engineering SAO algorithms. Examples include the very popular method of moving asymptotes (MMA) of Svanberg [2] (which is currently almost exclusively used in large topology optimization problems when multiple constraints are present), the well-known CONLIN algorithm of Fleury and Braibant [3], and the Dynamic-Q method of Snyman and Hay [4]. What is more, even optimality criterion methods (e.g., see Berke and Venkayya [5], Khot [6], Venkayya [7], and Rozvany [8]) lack a general proof of global convergence.

However, the foregoing does not mean that these algorithms cannot be rendered globally convergent. Two obvious strategies may be followed. The first is to cast these algorithms in a mathematical

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*Professor, Department of Mechanical and Mechatronics Engineering; albertg@sun.ac.za.

†Ph.D. Student, Department of Mechanical and Mechatronics Engineering; wderren@gmail.com.

‡Assistant Professor, Department of Mechanical Engineering, Post Office Box 513; l.f.p.etman@tue.nl. Member AIAA.

§Ph.D. Student, Department of Mechanical Engineering, Post Office Box 513; s.tosserams@tue.nl.