

# Interface Conditions of Finite Difference Compact Schemes for Computational Aeroacoustics

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**The interface condition provides flexibility to handle complex geometries on a set of structured grids. However, when used with a compact scheme to apply in computational aeroacoustics, its accuracy and stability may be degraded seriously by the boundary treatment that arises at a grid interface. To minimize this effect, a modified characteristic condition is incorporated in the finite difference compact scheme by using an explicit difference form at the boundary. The extension of this approach is naturally attained through the coordinate transformation of convective terms across an interface in multidimensions. The validity of current development is demonstrated through a simulation of trailing-edge noise generation from an airfoil at a low Mach number.**

## Nomenclature

$C_{\xi}$	=	modified convection term in the $\xi$ direction
$c$	=	convection velocity of a linear wave equation
$d_{\xi}$	=	$(x_{\xi}, y_{\xi})$
$\hat{E}$	=	transformed inviscid flux in the $\xi$ direction
$E, F$	=	inviscid fluxes in the $x$ and $y$ directions
$h$	=	width of a grid cell
$\text{Im}()$	=	imaginary part of a complex number
$J$	=	Jacobian matrix
$k$	=	wave number
$L$	=	one-dimensional domain length; the chord length of an airfoil
$M$	=	inflow Mach number
$\bar{P}^{-1}$	=	transformation matrix for characteristic decomposition
$p_{\infty}$	=	ambient pressure
$R$	=	characteristic differential variables
$\text{Re}()$	=	real part of a complex number
$r$	=	distance in the radial direction
$S_{\xi}$	=	reduced source term of the characteristic relation in the $\xi$ direction
$U_{\infty}$	=	inflow velocity
$u$	=	one-dimensional function $u(x, t)$
$\hat{u}$	=	Fourier coefficient of $u$
$u'$	=	spatial derivative of $u$
$u_o$	=	initial condition of $u(x, t)$
$\delta p_{\text{rms}}$	=	rms value of pressure fluctuation
$\kappa$	=	modified wave number
$\lambda$	=	eigenvalue of the Gustafsson-Kreiss-Sundström stability analysis
$\bar{\lambda}$	=	diagonal matrix of the speeds of characteristics
$\xi^L, \eta^L$	=	generalized coordinate on the grid $L$
$\xi^R, \eta^R$	=	generalized coordinate on the grid $R$

## I. Introduction

COMPACT finite difference schemes are now commonly used to achieve higher accuracy in many fields of computational analysis. Their low-dispersion nature is favored especially in the direct numerical simulations (DNS) or large-eddy simulations of

turbulent flows, as well as aeroacoustics, to resolve short wavelengths with high accuracy. One of the advantages of finite difference schemes is that their formulations can be extended easily to general coordinates. Recently, many application studies have been done on generalized curved grids using upwind or centered compact schemes. Also, the attempts to solve more complicated geometries are being made through multiblock approaches [1,2]. Usually, in a multiblock framework, the numerical grids must be carefully constructed to smoothly bridge grid topologies at a block connection, because the abrupt variation lowers the accuracy of flow simulations. With increasing geometrical complexity, however, it would be inhibitive to meet this requirement. On the other hand, Kim and Lee [3] introduced the characteristic interface conditions that do not necessarily require the smooth connection at a block interface, where the characteristic relations are applied from both sides; this approach has been extended to a more general treatment which alleviates the local-axis dependence across an interface [4]. Alternatively, to handle a complex geometry on a structured-grid basis, an overset-grid technique allows the use of high-order schemes as well: an entire flow domain is subdivided into a set of smaller components with a possibly simpler configuration (e.g., [5,6]). However, the characteristic interface condition would also increase flexibility in discretizing a subdivided region, not only limited in a multiblock framework. The purpose of the present article is to clarify how these interface treatments affect the accuracy of compact schemes, especially when used in computational aeroacoustics (CAA).

One of the difficulties in interface conditions is the boundary treatment of compact schemes at the interface. Because the derivatives are expressed implicitly in compact schemes, even if formally high-order schemes are used for inner nodes, actual accuracy may degrade seriously, which would result in unstable modes by using inappropriate approximations. Many studies have been conducted for this issue. For instance, Carpenter et al. [7] performed the semi-discrete eigenvalue analysis, originally developed by Strikwerda [8], on high-order boundary schemes, and showed that increasing the accuracy of boundary conditions would lead to instability. From the view point of the Fourier analysis, Sengupta et al. [9] pointed out that the boundary implementation can be critical to the stability of compact schemes, by also applying the von Neumann analysis, and derived a family of optimized upwind compact schemes. Also, to achieve more accurate schemes, optimization techniques were introduced to boundary closures [10,11], combined with centered compact schemes.

In this paper, our aim is to construct a proper formulation of characteristic-based interface conditions for the use with compact schemes. Here, we primarily concern the implementation of centered finite difference compact schemes, which is suited to aeroacoustic problems on, for example, a low-Mach-number flow, as well as turbulent flow. To develop a proper interface condition, the boundary

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