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Numerical Investigation of Decomposed Magnetofluid Dynamics Equations

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In addition to the two commonly used formulations for magnetofluid dynamics, a third formulation based on the decomposition of the magnetic field for solving full magnetofluid dynamics equations is explored. The governing equations are transformed to a generalized computational domain and discretized using a finite difference technique. A time-explicit multistage Runge–Kutta scheme augmented with total variation diminishing limiters for time integration is implemented. The developed codes have been validated with the existing closed-form solution of the magnetic Rayleigh problem. Results obtained from the decomposed magnetofluid dynamics equations compare well with results obtained by solving the classical full magnetofluid dynamics equations for a wide range of magnetic Reynolds numbers. It is shown that the decomposed magnetofluid dynamics technique requires substantially less computation time compared with classical full magnetofluid dynamics equations for the solution involving flowfields with high imposed magnetic fields.

Nomenclature

$\mathbf{B}$	=	magnetic field vector, $\begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix}$
$\mathbf{E}$	=	electric field vector, $\begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}$
$\mathbf{E}$	=	convective flux vector in the x direction
$\mathbf{E}_v$	=	diffusion flux vector in the x direction
e_t	=	total energy per unit mass
$\mathbf{F}$	=	convective flux-vector in the y direction
$\mathbf{F}_v$	=	diffusion flux-vector in the y direction
$\mathbf{G}$	=	convective flux vector in the z direction
$\mathbf{G}_v$	=	diffusion flux vector in the z direction
$\bar{\bar{\mathbf{I}}}$	=	identity tensor
J	=	Jacobian of transformation
$\mathbf{J}$	=	current density vector
M	=	Mach number
Pr	=	Prandtl number
p	=	pressure
$\mathbf{Q}$	=	field vector
Q	=	magnetic interaction parameter, $\frac{\sigma_e B^2 L}{\rho U}$
q	=	dynamic pressure
Re	=	Reynolds number
R_m	=	magnetic Reynolds number
t	=	time
$\mathbf{U}$	=	velocity vector, $\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$
x, y, z	=	Cartesian coordinates
∇	=	nabla operator
γ	=	ratio of specific heats
η, ξ, ζ	=	generalized coordinate
η_x, η_y, η_z	=	transformation metrics
μ_{eo}	=	free space magnetic permeability
ξ_x, ξ_y, ξ_z	=	transformation metrics
ρ	=	density
σ_e	=	electrical conductivity

Subscripts

e	=	electromagnetic quantity
i	=	induced magnetic field
o	=	imposed magnetic field
t	=	total magnetic field
v	=	diffusion quantity
∞	=	freestream condition

Superscript

n	=	iteration (time) level

Indices

i	=	index in the ξ direction
j	=	index in the η direction
k	=	index in the ζ direction

I. Introduction

SEVERAL numerical investigations have been dedicated to understanding the magnetofluid dynamics (MFD) of high-speed flows using classical full MFD (FMFD) and low magnetic Reynolds number approaches with different types of magnetic field distributions. Gaitonde and Poggie [1] used FMFD equations of finitely conducting fluid to simulate inviscid flow over a two-dimensional cylindrical body with nonuniform magnetic field distribution. An increase in shock standoff distance, a decrease in surface heat transfer, and a decrease in surface static pressure near the stagnation region were observed with the application of a magnetic field. The low magnetic Reynolds number approach was used by Poggie and Gaitonde [2] to model viscous and inviscid flows over a hemisphere. They illustrated that the application of a magnetic field caused an increase in shock standoff distance for both viscous and inviscid flows and concluded that qualitative changes in the pressure field obtained by applying a magnetic field were negligibly affected by viscous effects for simple blunt-body configurations.

The full MFD equations without Joule heating for two-dimensional blunt-body configurations was investigated by Hoffmann et al. [3]. Different types of magnetic field distributions were used; however, the maximum stand off distance for shock was observed for uniform

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