Dynamic Behavior of Angle-of-Attack Vane Assemblies

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Nomenclature

A = aspect ratio, w^2/S
b = semichord, i.e., chord = $2b$
$C_{L\alpha}$ = wing lift curve slope, $dC_L/d\alpha = \pi A/2$ for very small A
f_n = natural frequency in Hz, $\omega_n/2\pi$
$\dot{h}$ = transverse velocity of vane pivot axis, positive down
$\ddot{h}$ = transverse acceleration of vane pivot axis
J = mass rotary moment of inertia of the vane assembly
K = emperical stiction factor in dry friction term
$\mathcal{L}$ = distance between vane pivot axis and vane center of pressure
M = Mach number
q = freestream dynamic pressure
S = vane planform area
U = freestream velocity, usually $U_{EQV} = (2q/\rho_o)^{1/2}$
w = spanwise width
α = angular displacement with respect to freestream direction, positive clockwise
$\dot{\alpha}$ = angular velocity, $d\alpha/dt$
$\ddot{\alpha}$ = angular acceleration, $d^2\alpha/dt^2$
μ_D = dry friction coefficient
μ_V = viscous friction coefficient
π = 3.14159 . . .
ρ_o = sea level density of air
ζ = damping coefficient
ω_n = natural frequency, rad/sec

Introduction

RECENT events in flight tests have generated the need for a verified model of the dynamic behavior of angle-of-attack vanes. For example, dynamic performance flight testing[1] is being considered to reduce the duration and cost of test programs. Also, angle-of-attack vanes have been used to obtain gust data in thunderstorm penetration studies.[2] Extraneous angular motions due to vibration of the vane's support (usually a somewhat flexible boom) are a further concern.

Dynamic Model

A mathematical model of a vane's dynamic behavior was developed based on the equations of motion of a plunging, rotating, two-dimensional flat plate in an inviscid, incompressible fluid.[3] Combining the aerodynamic moments [for example, from Eq. (5-348) of Bisplinghoff et al.[4]] with the vane's rotary inertia and some viscous and dry friction terms to account for mechanical damping due to bearings and the like, one can obtain

$$\ddot{\alpha} + (2\zeta\omega_n + \mu_v)\dot{\alpha} + \omega_n{}^2\alpha + \mu_d \min(K\dot{\alpha}, \operatorname{sgn}\dot{\alpha}) = -\frac{\omega_n^2}{U}\left[\dot{h} + \frac{(2\mathcal{L}+b)}{4\mathcal{L}}\;\frac{b}{U}\ddot{h}\right] \tag{1}$$

where

$$\omega_n = \left\{C_{L\alpha}\mathcal{L}q\,S/\left[J + \left(2\frac{\mathcal{L}}{b}+1\right)^2 \frac{\pi}{8}\rho_o b^3 S\right]\right\}^{1/2} \tag{2}$$

and

$$\zeta = \left[(2\mathcal{L}^2 + 3\mathcal{L}b + b^2)/4\mathcal{L}^2\right]\left[\mathcal{L}/U\right]\omega_n \tag{3}$$

For typical vanes (relatively small inertias) and reasonable airspeeds (greater than 50 mph), the frictional terms should become negligible as do the $\ddot{h}$ term and the aerodynamic inertia term, so these equations simplify to

$$\ddot{\alpha} + 2\zeta\,\omega_n\dot{\alpha} + \omega_n{}^2\alpha = -\omega_n{}^2(\dot{h}/U) \tag{4}$$

where

$$\omega_n = (C_{L\alpha}\mathcal{L}\,q\,S/J)^{1/2} \tag{5}$$

and ζ is still given by Eq. (3). The key assumption made was that the two-dimensional predictions of aerodynamic moments would still be valid if the finite wing lift curve slope $C_{L\alpha}$, and the actual distance from the pivot point to the center of pressure $\mathcal{L}$ were used in lieu of their infinite wing counterparts. While Eqs. (3) and (5) have previously been developed[5,6]. . . at least as the limiting value of $\zeta = (\mathcal{L}/2)(\omega_n/U)$ for $\mathcal{L} \gg b$, this more complete form provides guidance as to the second order terms neglected in more straight-forward developments.

In applying this model, several comments are in order. First, if expressed in terms of equivalent airspeed at sea level, the damping is independent of airspeed. Secondly, since vanes are invariably of very low aspect ratio, the slender body $C_{L\alpha} = \pi A/2$ should be used. Unfortunately, predicting the center of pressure location for such low aspect ratios is difficult at best. An extrapolation of the results of Table 1 of Gersten[7] may be used for rectangular vanes, while the slender body prediction of Jones[8] may be used for triangular or delta vanes. Fortunately, Jones'[8] slender body theory also may infer that this model would be valid for all airspeeds (except maybe transonic) in spite of its incompressible development.

Experimental Verification

A rectangular, flat plate vane used by the 4950th Test Wing at Wright-Patterson AFB for thunderstorm penetration studies was dynamically tested in the Air Force Institute of Technology 5 ft diam, low subsonic wind tunnel. The vane assembly was mounted on the upstream end of the horizontal member of a cruciform mounting assembly. The vane is a fiberglass and balsa rectangle pivoted about its leading edge. Its chord ($2b$) is 4.75 in. and its aspect ratio is 0.5. A steel cylindrical counterweight is attached so that the vane is statically balanced. The vane assembly's inertia was calculated and experimentally verified to be 0.0012 in lb$f$ sec.[2]

An initial series of tests were run with the cruciform stationary ($\dot{h} \equiv 0$), and the vane was released from an initial angle of attack of about 5°. The upper part of Fig. 1 is a typical response. As expected, the response is essentially that of a damped second-order system. The test results are shown in Fig. 2 which also includes the predictions of the model. The damping coefficient was determined from the log decrement

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Index categories: Aircraft Testing (including Component Wind Tunnel Testing); Research Facilities and Instrumentation.

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