Improved Thin-Airfoil Theory

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The classical thin-airfoil theory has been reconsidered with the purpose of improving its accuracy and extending its range of applicability. Two improvements were implemented. First, the effect of profile thickness on the overall aerodynamic parameters C_L and C_M has been allowed for. The usual expressions for these aerodynamic parameters are extended by new corrective terms and the closed-form nature is thus preserved. Second, a simple procedure has been developed to determine the coefficients in the series expansions of the vortex and source distributions by directly using the given profile coordinates instead of the less accurate numerically determined slopes. Over a wide range of Karman-Trefftz airfoil geometries and angles of attack with only five control points, it was found that the accuracy of the present method is substantially improved and its validity is extended to thickness ratios as high as 16%. Surprisingly, except for very limited situations, the accuracy of the method was found to be comparable to or even better than that of the Hess-Smith method using 100 panels.

I. Introduction

SINGULARITY methods are among the most widely used techniques in solving inviscid flowfields around airfoils. The basic idea is to simulate the airfoil by one or more types of continuously distributed or discrete singularities. This principle is quite old and can be traced back to Prandtl and his colleagues at Goettingen in the period 1912–1922.[1] Two approaches for applying this principle are available: the classical thin-airfoil theory and the surface singularity or panel approach. The latter is more powerful, but its computational complexity is enormous. In fact, its implementation had to await the recent development of large high-speed computers. Since then, a large number of panel methods have been developed by employing different types of singularities and following slightly different schemes in evaluating their strength.[2-6] On the other hand, the classical thin-airfoil theory is an approximate method limited to thin, slightly cambered airfoils operating at small incidence. Nevertheless, it is attractive for two main reasons: 1) it decouples the camber, thickness, and incidence effects whose contributions then simply become additive; and 2) the solution is obtained in analytic form using simple trigonometric series and can easily be performed on a programmable calculator.

In this classical thin-airfoil theory, the solution is split into two parts. The first combines the camber and incidence problems and completely ignores thickness. The thin airfoil, now represented by its camberline, is simulated by a vortex sheet. The vortex strength distribution $\gamma(x)$ is calculated by satisfying the tangency condition on the camberline using v/V_∞ induced on the chord line and the Kutta condition at the trailing edge. The second part allows for the thickness effect and ignores camber and incidence altogether. This reduces the solution of the thickness problem to that of the flow past a symmetrical airfoil at zero incidence and yields equal velocity contributions on the lower and upper sides. In the relevant literature on the subject, there is very little quantitative information about the range of applicability of the thin-airfoil theory beyond which its accuracy deteriorates. One of the few quoted limits is found in Anderson,[1] who globally states that the method gives "good" results for thickness ratios up to 12%.

In the present study, we set forth two objectives. The first is a detailed parameteric study to outline quantitatively the range of applicability of the classical thin-airfoil theory. The second objective is more ambitious and aims at improving the accuracy of the method and extending its range of applicability. This is achieved by satisfying the boundary conditions using the profile coordinates rather than the slopes in both the camber and thickness problems and by accounting for the effect of thickness on the lift and moment coefficients. The results of this extended thin-airfoil method are compared with those of the classical thin-airfoil theory and with the exact solution, over a wide range of thickness ratios, camber ratios, trailing-edge angles, and angles of attack, for a family of Karman-Trefftz airfoils. The results of the Hess-Smith panel method are also included in these comparisons.

II. Previous Work

The classical thin-airfoil theory is well documented in most aerodynamics texts;[1,5,7] therefore, it is sufficient to outline here only the main features needed to build on in subsequent sections.

With reference to Fig. 1, the tangency condition can be approximated, for small angles of attack and camber, by

$$\frac{dy}{dx} = \alpha - \frac{1}{2\pi V_\infty}\int_0^\ell \frac{\gamma(\xi)\,d\xi}{x-\xi} \tag{1}$$

where dy/dx is the slope of the camber line and γ the local vortex strength. A rigorous solution of integral equation (1) that satisfies the Kutta condition ($\gamma_{TE} = 0$) has been found to be

$$\frac{\gamma(\theta)}{2V_\infty} = A_0 \cot\frac{\theta}{2} + \sum_{i=1}^{\infty} A_i \sin i\theta \tag{2}$$

where θ is related to the dummy variable ξ by the transformation $\xi/\ell = 0.5\,(1-\cos\theta)$. Substituting Eq. (2) into Eq. (1) and integrating, one obtains

$$\frac{dy}{dx} = \alpha - A_0 + \sum_{i=1}^{\infty} A_i \cos i\phi \tag{3}$$

where ϕ is related to x in the same way θ is related to ξ. Using simple Fourier analysis, the vortex coefficients are shown

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