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Optimized Measurements of Unmanned-Air-Vehicle Mass Moment of Inertia with a Bifilar Pendulum

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A bifilar (two-wire) pendulum is a torsional pendulum consisting of a test object suspended by two thin parallel wires. The pendulum oscillates about the vertical axis. The restoring torque of the bifilar pendulum is provided by the gravitational force as rotations from the rest state cause the test object to raise slightly. The mass moment of inertia is computed using dynamic modeling, measurements of the oscillation period, and the physical dimensions of the bifilar pendulum such as the length and separation displacement of the pendulum wires. A simulation technique is described that improves estimates of the mass moment of inertia by considering the nonlinear effects of damping and large angular displacements. An analysis of the error variance of mass moment of inertia measurements is also described. The resulting expression for the error variance is used to optimize the physical parameters of the bifilar pendulum to obtain the moment of inertia measurement with the minimum error variance. Monte Carlo simulations were used to validate the parameter optimization technique. Experimental results are presented for a uniform-density test object for which the moment of inertia is straightforward to compute from geometric considerations. Results are also presented for a small unmanned air vehicle, which was the intended application for this moment of inertia measurement technique.

I. Introduction

MEASUREMENTS or computational estimates of mass moment of inertia are needed during the design and construction of aircraft, including unmanned air vehicles (UAVs), which have received sustained interest in recent years. The bifilar pendulum is an apparatus that has been used for the measurement of aircraft mass moments of inertia since near the time of the invention of the airplane because of its simplicity, safety, and relatively high accuracy [1]. The bifilar pendulum, shown in Fig. 1, is a torsional pendulum that consists of a test object suspended by two thin parallel wires of length h and separation displacement D. The pendulum oscillates about its vertical axis. A tare platform is often used to help configure the test object and to provide a tare mechanism for improving measurement accuracy. The rotation angle in the horizontal plane is θ. The restoring torque of the bifilar pendulum is provided by the gravitational force as rotations from rest cause the test object to raise slightly. Dynamic modeling is used to relate the measurable parameters of average oscillation period, wire length, and wire separation displacement to the moment of inertia of the test object.

In the early experimental work by the National Advisory Committee for Aeronautics (NACA) that measured mass moment of inertia of small manned aircraft with a bifilar pendulum, the common technique was to ignore damping and to linearize the equations of motion to model the bifilar pendulum as a harmonic undamped oscillator [1,2]. The bifilar pendulum was used for the measurement of moment of inertia about the yaw axis. Although some reasoning was used to set apparatus parameters, the choice of specific parameters was not made based on any rigorously derived criteria, and there was no reported effort made to determine quantitative measurement errors due to apparatus dimensions.

In later work, damping was still neglected, but researchers began to incorporate corrections to account for momentum transfer between the pendulum and the surrounding air [3]. These corrections become important for test objects with large surface areas such as fixed wing aircraft, particularly when rotating about the roll axis. The motion of these objects entrains some of the air through which they rotate. The entrained air is most accurately modeled as an additional mass of the system rather than as a damping effect. This effect commonly appears in the literature on modeling and control of underwater manipulators where fluid momentum effects account for a significant portion of external forces on a body [4].

A detailed nonlinear model of the bifilar pendulum was developed by Kane [5], primarily to examine the effects of uneven pendulum geometries. The nonlinear model was used to show that torsional motions of the bifilar pendulum are not significantly affected by uneven pendulum wire lengths or misaligned principal axes. Damping effects were not examined and the selection of apparatus dimensions was not discussed.

An excellent description of prior work on the measurement of mass moments of inertia for aircraft is found in [6]. A method for estimating multiple inertia parameters from a single experiment using statistical estimation techniques is also described. Aerodynamic damping was accounted for in the development of the equations of motion, but not viscous damping. The equations of motion were then linearized to develop the estimation algorithms. Additional mass corrections were noted, but not addressed, as the main objective of the paper was to present a simplified procedure for measuring moment of inertia and not necessarily to improve the accuracy of the measurements.

The authors of [7] examined various hardware improvements and methodologies for improving modeling accuracy in a trifilar, or three-wire, pendulum. Errors due to linearization were mitigated by making accurate rig-tare measurements using objects with known moments of inertia. Pendulum precession caused inaccurate measurements of the pendulum oscillation period because direct measurements of the period were made using an optical sensor.

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