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Unified Flutter Solution Technique Using Matrix Singularity Indicators

Zhichun Yang* and Yingsong Gu†
Northwestern Polytechnical University, 710072 Xi'an, People's Republic of China

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Conventional linear flutter analysis using the *k* or *p-k* method is based on eigenvalue solution of the flutter equation, which needs to track the eigenmodes to give a proper flutter point, but sometimes the procedure may fail. As a step toward a flutter solution method with more automation and robustness, a discussion is made for the uniqueness of the flutter matrix singularity at and inside the flutter boundary, and proper clearance procedure against nonuniqueness is provided. Four indicators of matrix singularity are introduced: that is, the determinant module, minimum module eigenvalue, minimum singular value, and inverse condition number of the flutter matrix. Finally, a unified algorithm is developed for flutter solution with these indicators. Numerical examples demonstrated that the results by the present method coincide well with the *p-k* and *k* methods but need no mode tracking. It is also shown that the coupled flutter-mode information can be extracted qualitatively from frequency diagrams of these indicators at different velocities and quantitatively from the eigenvector corresponding to the minimum module eigenvalue of the flutter matrix at the critical flutter point.

Nomenclature

$\boldsymbol{B}$	=	generalized damping matrix
b	=	semichord, ft
$\boldsymbol{K}$	=	generalized stiffness matrix
k	=	reduced frequency
$\boldsymbol{M}$	=	generalized mass matrix
$\boldsymbol{\eta}$	=	generalized displacement coordinates
n	=	number of modes incorporated in the aeroelastic model
$\boldsymbol{Q}$	=	generalized aerodynamic influence coefficient matrix
q_∞	=	dynamic pressure, $1/2\rho_\infty V^2$
s	=	Laplace variable
V	=	velocity, ft/s
V_F, V_f	=	theoretical and calculated flutter speed, ft/s
ρ_∞	=	air density, lb/in.^{3}
ω	=	frequency, rad/s
ω_F, ω_f	=	theoretical and calculated flutter frequency, rad/s

Introduction

LINEAR flutter analysis is now very mature in the aeroelastic research field. There are two classical flutter solution methods: namely, the k method developed by Theodorsen [1] and the p-k method by Hassig [2]. Both are widely used in aeronautical engineering and well documented in texts of aeroelasticity [3,4], but the p-k method is usually favored because it can give a damping factor that is physically meaningful, as opposed to the artificial damping term introduced by the k method. In recent years, a true damping solution technique, called the g method, was proposed by Chen [5], which can obtain unlimited roots. The p-k method has to solve for the eigenvalues of flutter equation iteratively and it tracks eigenmodes to give the proper flutter result. Such a procedure may become very critical when two aeroelastic modes have quite close frequencies, which is sometimes called the branch-crossing problem [6]. The situation can be quite difficult in real-world problems, especially the multidisciplinary-optimization (MDO) environment [7], in which mode tracking is not always effective and human interaction is usually needed to validate the results such as the V–g diagrams. This is tedious work and may make the optimization an impossible job.

Afolabi [8] developed a new flutter analysis approach using transversality theory with mathematical insight to aeroelastic instability. According to Afolabi's theory, the discriminant of the characteristic polynomial vanishes at the flutter point with the occurrence of degenerate eigenvalues. This conclusion strictly depends on the expression of steady aerodynamics, because the discussion is limited to the characteristic polynomial with all real coefficients, but unsteady aerodynamics employed for general wing surface are always complex in the frequency domain. Hence, the discriminant may not be zero in such cases.

It is found for specific aeroelastic stability problems, such as panel flutter in supersonic flow, that the orientations of flutter eigenvectors can be used to predict the flutter point; namely, the angle between two eigenvectors decreases to zero and hence indicates flutter [9]. More recently, this approach was employed in flutter prediction for control law design in a flutter suppression study with piezoelectric layers [10]. However, this method may suffer the same problem as that described in [8] in general aeroelastic applications, such as wing or tail flutter solutions.

The preceding flutter solution methods are based on either eigenvalue or eigenvector orientations, and there are still other approaches using only matrix singularities. Matrix singular value approach was demonstrated as a powerful tool in the multivariable feedback system stability analysis [11] and it was later applied to study the stability margin of a multiloop flutter suppression system [12]. Furthermore, it was also adopted to evaluate the performance of control system in companion with the determinant and minimum module eigenvalue of the return difference matrix [13,14]. The determinant of the flutter matrix was introduced as a flutter indicator by Banerjee [15], and it was later integrated into an aeroelastic optimization routine [16]. The concept is very insightful; that is, the flutter determinant vanishes when flutter occurs.

Singularity of the flutter matrix is a global indicator of the flutter point, which is independent of any specific mode explicitly. In this effort, a discussion is first conducted to ensure uniqueness of such singularity at and inside the flutter boundary with proper clearance procedure against nonuniqueness problems, which makes it possible to develop a simple and robust flutter solution technique without branch-crossing problems. Four singularity indicators are presented

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*Professor, School of Aeronautics, P.O. Box 118, Youyi Xilu No. 127; yangzc@nwpu.edu.cn. Member AIAA.

†Ph.D. Candidate, School of Aeronautics, P.O. Box 118, Youyi Xilu No. 127. Student Member AIAA.