JOURNAL OF AIRCRAFT
Vol. 46, No. 5, September–October 2009

Numerical Evaluation of Limit Cycles of Aeroelastic Systems

Mauro Manetti,* Giuseppe Quaranta,† and Paolo Mantegazza‡
Politecnico di Milano, 20156 Milan, Italy

DOI: 10.2514/1.42928

This paper focuses on the analysis of limit-cycle oscillations of aeroelastic systems with multiple lumped nonlinearities. It aims at a comprehensive investigation capable of identifying limit cycles and their stability. The goal is achieved by using an incremental complexity approach. At the beginning, a solution based on dual-input describing functions is sought, to find both symmetric and asymmetric cycles approximated to their first harmonic. The related stability is investigated afterward by extending the single-input describing function "quasi-static" method. Such an approach is simple and quite similar to well-established existing methods used to evaluate linear flutter conditions directly. If higher harmonics are required, an extended harmonic balance based on a numerical minimization in the frequency domain is adopted and the stability of the computed solutions is then determined by using Floquet theory. The presented approach is applied to several nonlinear aeroelastic examples and validated by comparing stable limit cycles with solutions obtained through direct time marching integrations.

Nomenclature

$\boldsymbol{A}_a, \boldsymbol{B}_a, \boldsymbol{C}_a$ = aerodynamics state-space matrices
$\boldsymbol{a}$ = aerodynamic state vector
$\arg(\cdot)$ = phase of a complex number, $\tan^{-1}(\Im/\Re)$
$\bar{\boldsymbol{B}}_{\mathrm{nl}}$ = input matrix for lumped nonlinearities on structural degrees of freedom
$\boldsymbol{b}_i$ = real Fourier coefficients
$\boldsymbol{C}$ = structural damping matrix
$\boldsymbol{c}_i$ = imaginary Fourier coefficient
$\boldsymbol{D}_0, \boldsymbol{D}_1, \boldsymbol{D}_2$ = aerodynamics quasi-steady matrices
$\det(\cdot)$ = matrix determinant
$\mathrm{diag}(\cdot)$ = diagonal matrix
$\boldsymbol{F}_a$ = generalized aerodynamic forces, Nm
G = transfer function
H_n = Fourier coefficient $\in \mathbb{C}$
h = plunge displacement
$\boldsymbol{I}$ = unit matrix
$\Im[\cdot]$ = imaginary part of a complex number
J = moment of inertia, kgm^2
j = imaginary unit
$\boldsymbol{K}$ = structural stiffness matrix
l_a = reference length, m
$\boldsymbol{M}$ = structural mass matrix
M_∞ = Mach number
m = mass, kg
N = describing function coefficients $\in \mathbb{C}$
$\boldsymbol{Q}$ = transition matrix
$\boldsymbol{q}$ = generalized structural degrees of freedom
q_∞ = dynamic pressure, Pa
$\Re[\cdot]$ = real part of a complex number
r = radius of gyration
S = first mass moment, kgm
s = complex variable
T = period, s
$\boldsymbol{u}$ = structural nodes displacement vector
V_∞ = reference velocity, m/s
$\boldsymbol{x}$ = spatial coordinates
z = global state vector
α = pitch rotation, rad
α_{lc} = limit-cycle scalar amplitude
β_i = displacement of the ith lumped nonlinearity
$\dot{\beta}_i$ = velocity of the ith lumped nonlinearity
$\boldsymbol{\gamma}$ = describing function bias vector
$\boldsymbol{\delta}$ = describing function amplitude vector
$\boldsymbol{\varepsilon}$ = residual vector
η_i = complex Fourier coefficients
μ = mass ratio
ρ = air density, kg/m^3
ζ_i = real Fourier coefficients
$\boldsymbol{\Phi}$ = structural shape functions
ω = circular frequency, rad/s

Subscripts

d = dual input
h = plunge
lc = limit cycle
nl = nonlinear
ql = quasi linearized
α = pitch
β = flap

I. Introduction

ROUTINE flutter analyses are carried out mostly on linearized models, both for the structure and the aerodynamics. They provide results that are often in good agreement with test outcomes. Nonetheless, the assumption of negligible nonlinearities may sometimes hide problems revealed only during test and certification phases.

In general, nonlinearities in the fluid and/or the structure result in complex system behaviors, such as limit-cycle oscillations (LCOs), that are of much interest in aircraft design, because system stability may become dependent on the amplitude of the disturbance. Using a nonlinear dynamics point of view, the crossing of the flutter onset leads to a Hopf bifurcation where a periodic self-sustained motion, the LCO, branches from a given equilibrium solution [1]. The frequency and amplitude characteristics of the LCO depend on the dynamic system properties only.

The first studies on structural lumped nonlinearities in aeroelasticity began in the 1950s [2,3]. Nonlinearities were used to represent free play of mechanisms, friction in bearings, cables, bars, and servo-actuator systems. Aerodynamic nonlinearities may also

Received 25 December 2008; revision received 14 May 2009; accepted for publication 18 May 2009. Copyright © 2009 by Politecnico di Milano. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. Copies of this paper may be made for personal or internal use, on condition that the copier pay the $10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0021-8669/09 and $10.00 in correspondence with the CCC.

*Ph.D. Candidate, Dipartimento di Ingegneria Aerospaziale, Via La Masa, 34; manetti@aero.polimi.it.

†Postdoctoral Research Fellow, Dipartimento di Ingegneria Aerospaziale, Via La Masa, 34; giuseppe.quaranta@polimi.it.

‡Full Professor, Dipartimento di Ingegneria Aerospaziale, Via La Masa, 34; paolo.mantegazza@polimi.it.