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Role of Invariant Manifolds in Low-Thrust Trajectory Design

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This paper demonstrates the significant role that invariant manifolds play in the dynamics of low-thrust trajectories moving through unstable regions in the three-body problem. It shows that an optimization algorithm incorporating no knowledge of invariant manifolds converges on low-thrust trajectories that use the invariant manifolds of unstable resonant orbits to traverse resonances. It is determined that the algorithm could both change the energy through thrusting to a level where the invariant manifolds could more easily be used, as well as use thrusting to move the trajectory along the invariant manifolds. Knowledge of this relationship has the potential to be very useful in developing initial guesses and new control laws for these optimization algorithms. In particular, this approach can speed up the convergence of the optimization process, retain the essential geometric and topological characteristics of the initial design, and provide a more accurate estimate of the ΔV and fuel usage based on the initial trajectory.

Introduction

The use of low thrust in trajectory design can significantly increase the complexity of the design process, because many of the standard astrodynamics tools are no longer applicable without what are sometimes significant modifications. For modeling performed in the two-body problem, the use of low thrust increases the difficulty of design in that the resulting trajectory no longer follows conic sections. In the three-body problem, the energy, or the Jacobi constant, changes because of the continuous thrust. As a result of these difficulties, much of the design work for low-thrust missions is performed using optimization tools which do not necessarily incorporate a full knowledge of the dynamics of the problem in the search for a desired trajectory. It has been observed, however, that the solutions developed using the Mystic optimization software [1–3] appear to generally follow the same types of paths as the invariant manifolds of unstable periodic orbits in the three-body problem [4]. This suggests that a knowledge of the relationship of these optimized trajectories to the invariant manifolds of unstable orbits could prove to be useful in the design of low-thrust trajectories. The long periods of time often required to run the optimization software could be significantly reduced if a good initial guess could be developed using the dynamics of the problem based on the invariant manifolds of the relevant unstable orbits. In this paper, we demonstrate how invariant manifolds play a central role in optimized low-thrust trajectories.

This paper arose from a study to understand optimized low-thrust trajectories for multimoon tours from a dynamical systems perspective. It specifically concentrates on the relationship of optimized low-thrust trajectories to the invariant manifolds of resonant orbits. The results given in this paper summarize the low-thrust results originally presented in a series of papers from 2004 to 2006 [5–7]. A more detailed version of these results may be found in Anderson's dissertation (2005) [8]. Our stated goals in this work have been to demonstrate that invariant manifolds do indeed play a role in low-thrust trajectories and to explain how the dynamics of low-thrust interplanetary trajectories interact with invariant manifolds. Lo et al. [5] compared low-thrust trajectories to the invariant manifolds of nearby unstable orbits at a single energy level. The outcome of this work suggests heuristically that we are on the right track, but, because low-thrust trajectories are constantly changing their Jacobi energy while thrusting, one must study a continuum of invariant manifolds in the energy range of the low-thrust trajectory. To do this, we must first understand the role of resonant orbits in planetary flybys and whether invariant manifolds play a role or not. For these reasons, in Anderson and Lo [6], we analyzed the planar Europa Orbiter (PEO) trajectory and found that it indeed follows the stable and unstable manifolds of the resonant orbits between impulsive maneuvers. In particular, it was found that the locations where the manifolds intersect in configuration space but not in phase space are the locations where a maneuver is required for moving from one manifold to another. This result suggests that a deeper understanding of the geometry of the invariant manifolds of resonant orbits is critical to understanding planetary flybys, and we anticipate this to also be true for low-thrust trajectories when they are moving through resonant orbit regions. These conjectures were found to be true, and the close relationship between low-thrust trajectories and the invariant manifolds of unstable resonant orbits was shown in Lo et al. [7]. Since that time, subsequent research has been performed by several groups building on the optimization techniques developed by Lawden [9], Betts and Erb [10], and Betts [11]. In addition to this work, several of these researchers have begun to focus on optimization within a multibody environment, particularly with regard to using libration point orbits. Whiffen and Lam have continued to develop and apply the Mystic software to missions such as the Jupiter Icy Moons Orbiter [12] and the Dawn mission. Howell's group has applied optimization techniques to trajectories with gravity flybys and looked at low-thrust trajectories transferring to libration point orbits in the Earth–moon system [13,14]. Dellnitz et al. used the concept of reachable sets in combination with the invariant manifolds of libration orbits to look at a low-thrust transfer from Earth to Venus [15].

Studying low-thrust trajectories in this type of environment continues to be a vibrant area of research, and this fact emphasizes the importance of continuing to explore these trajectories in a three-body environment. The role of resonance transition that our papers have focused on continues to be important for understanding trajectories within the context of this problem. In this paper, we examine and summarize the relationship between a low-thrust trajectory and the invariant manifolds of several families of resonant orbits through the energy levels traversed by the low-thrust trajectory.

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