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Indirect Optimization of Three-Dimensional Finite-Burning Interplanetary Transfers Including Spiral Dynamics

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The indirect optimization problem for a three-dimensional transfer from low Earth orbit to low Mars orbit is solved. A step-by-step process developed for a two-dimensional model and techniques for accurately estimating the unknown costates for three-dimensional escape and capture spirals are used. Minimum-propellant trajectories for finite-burning engines are calculated. Solutions are considered with and without control limits on specific impulse and compared with previous research. Unlike other research, the entire trajectory, including the Martian capture sequence, is integrated in an Earth-referenced frame. Additionally, the capture sequence is not found by iteratively lowering the final targeted low Mars orbit, but the desired final orbit is directly targeted with no successive iterations of increasingly smaller low Mars orbits. As in the two-dimensional case, more fuel-efficient trajectories are found for the same mission objectives and constraints published in other research, emphasizing the importance of this technique. Whereas previous research only achieved final Martian orbits of 6 Mars radii DU_M (20,382 km), the new approach finds solutions for final Martian circular orbits of 1.47–2.00 DU_M (5000–6794 km).

Nomenclature

$\mathbf{A}$	=	rotation matrix from the heliocentric equatorial frame to the Earth-centered rotating frame in three dimensions
A', B', C'	=	auxiliary variables used to express the areocentric spacecraft position in the Earth-centered rotating coordinates
$\mathbf{AB}$	=	rotation matrix from the Mars-centered rotating frame to the Earth-centered rotating frame in three dimensions
$\mathbf{ABd}$	=	time derivative of $\mathbf{AB}$
a	=	thrust-acceleration magnitude
$\mathbf{B}$	=	rotation matrix from the Mars-centered rotating frame to heliocentric equatorial frame in three dimensions
ECR	=	spacecraft-centered spherical frame rotating with Earth around the sun
$\mathbf{EC2ECR}$	=	rotation matrix from the Earth ecliptic frame to the Earth-centered rotating frame in three dimensions
$\mathbf{EC2EQ}$	=	rotation from the Earth ecliptic frame to the heliocentric equatorial frame in three dimensions
$\mathbf{EC2EQM}$	=	rotation from the Martian ecliptic frame to the heliocentric equatorial frame in three dimensions
$\mathbf{EC2RE}$	=	rotation from the Earth ecliptic frame to the Earth planetary spherical frame in three dimensions
$\mathbf{EQ2EC}$	=	rotation from the heliocentric equatorial to the Earth ecliptic frame in three dimensions
$\mathbf{EQ2ECM}$	=	rotation from the heliocentric equatorial to the Martian ecliptic frame in three dimensions
$\mathbf{e}_*$	=	unit vector for coordinate frames
G	=	Bolza function
H	=	Hamiltonian
J	=	cost function
MCR	=	spacecraft-centered spherical frame rotating with Mars around the sun
$\mathbf{MCR2RM}$	=	rotation from the Mars-centered rotating frame to the Mars planetary spherical frame in three dimensions
$\mathbf{MCR2EC}$	=	rotation from the Mars-centered rotating frame to the Mars ecliptic frame in three dimensions
m	=	spacecraft mass
P	=	spacecraft power
$\mathbf{R}_s$	=	position vector of the spacecraft from the sun
$\mathbf{R}_*$	=	position vector of planet $*$ from the sun
$\mathbf{RE2ECR}$	=	rotation from the Earth planetary spherical to the Earth-centered rotating frame in three dimensions
$\mathbf{RM2MEC}$	=	rotation from the Martian planetary spherical to the Martian ecliptic frame in three dimensions
r, θ, Φ	=	spherical position components
$\mathbf{r}$	=	position vector in the Earth-centered rotating frame
r_r, r_θ, r_Φ	=	position of the spacecraft ($\mathbf{r}$) with respect to Earth in the Earth planetary spherical frame
t	=	time
$\mathbf{u}$	=	thrust-acceleration unit direction vector
V_{RS}, $V_{\theta S}$ $V_{\Phi S}$	=	inertial heliocentric velocity components (S subscript) of the spacecraft in the Earth-centered rotating frame (R, θ, and Φ subscripts) axes
V_r, V_θ, V_Φ	=	spherical velocity components of the spacecraft in the Earth-centered and Mars-centered rotating frames
V_{*r}, $V_{*\theta}$	=	polar velocity components of planet $*$ around the sun
λ	=	costates
μ	=	gravitational parameter
$\boldsymbol{\omega}$	=	costate vector adjoined to constraints/targets in the Bolza function

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