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Structure Preserving Approximations of Conservative Forces for Application to Small-Body Dynamics

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Approximation-based methods, such as the cubetree algorithm, have proven to be significantly faster than traditional methods for complex force evaluations near small irregular bodies. Such methods also hold the promise of simplifying the inclusion of experimental data to update the force model. However, the cubetree algorithm does not preserve intrinsic properties of the gravitational force such as continuity, divergence freedom, or exactness. These properties may be needed for trajectory optimization, for the use of geometric (e.g., symplectic) integrators for long-term propagation, and for other trajectory design problems. This paper presents several adaptive schemes preserving global continuity, exactness, or divergence freedom, and discusses the difficulties involved in preserving all of these properties globally.

I. Introduction

WITH the continuing increase in computing power, large-scale problems, which were considered very difficult in the past decade, have now become tractable. In particular, optimization methods based on genetic algorithms have recently attracted many researchers in astrodynamics [1,2], and simulations involving hundreds of thousands of trajectory propagations have recently appeared [3,4]. However, there is still a need to improve the core algorithms, such as ordinary differential equation integration and force function evaluation, for several reasons.

First, faster elementary methods mean that larger, more realistic systems can be considered. In particular, large simulations generally assume fairly simple dynamics and are more challenging for complex force models such as small-body environments. Second, as autonomous navigation becomes a reality, there is an increased demand for fast onboard computational tools [5]. Finally, current research in numerical integration emphasizes the importance of preserving fundamental geometric structures present in the modeled dynamics. Such issues have appeared to be of prime importance for long-term integration, such as encountered in astronomy and in the analysis of numerical experiments.

Recently, a novel numerical scheme (the cubetree algorithm) for the fast evaluation of gravitational force around irregular bodies has been introduced by the authors [6] and shown to provide a significant speed improvement over other methods. The scheme exploits the availability of large storage capacities to reduce the online computations. Specifically, by locally interpolating the force field around a small body, this algorithm decreased computational effort of spacecraft trajectories integration by a factor of 100. Although such results are of particular interest for large simulations, such as Monte Carlo analysis, the method may also be useful for smaller, onboard computation due to its relatively light load on the processor. However, the method has not been optimized and requires a significant memory footprint. Also, several desirable mathematical properties of gravitational force have not been considered and are not preserved by the cubetree algorithm.

In this paper some of these issues are addressed by developing improved approximation schemes for potential energy and force representation around small irregular bodies. In particular, the following questions are considered.

Smoothness: When several interpolation domains are considered, discontinuities at the boundary represent a fundamental obstacle for theoretical investigation due to the continuous nature of the force represented. These discontinuities may also lead to a deterioration of integration performance.

Exactness: All conservative forces are the gradient of some potential field. The cubetree method presented in [6] does not respect this qualitative feature. This is especially important for long-term simulation where qualitative features of the trajectories are of primary interest.

Divergence freedom: The force of gravitation is divergence free. This property has many theoretical implications, but is not necessarily preserved by standard interpolation schemes.

Efficiency: What interpolation schemes allow for smaller memory footprint while ensuring sufficient accuracy? This question may be addressed in both the approximation method and the choice of subdivision technique used for partitioning the space around the body.

Note that addressing the first two issues is a necessary step for the application of geometric integrators such as symplectic integrators. Although geometric integrators have been applied to problems with discontinuities and dissipation, the structure of discontinuities or dissipation is part of the theoretical framework in those cases [7]. In an approximation scheme, like the cubetree method, it is due to the approximation of the force field.

To tackle the aforementioned issues, several modifications of the cubetree algorithm have been considered. In Sec. III, a regularization of cubetree that produces a continuous approximation is presented. While easily implementable, this solution requires tighter tolerances on the approximated function, which increases the memory footprint of the model. Sec. IV presents a different approach based on least-squares approximation using hierarchical B-spline refinements, which provided a less stringent error requirement while addressing

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