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# Guidance Laws with Finite Time Convergence

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**Conventional guidance laws are designed based on Lyapunov theorems on asymptotic stability or exponential stability. They will guide the line-of-sight angular rate to converge to zero or its small neighborhood, however, only as time approaches infinity. In this paper, new guidance laws with finite convergent time are proposed. The guidance laws are obtained based on new sufficient conditions derived in this paper for the finite time convergence of the line-of-sight angular rate. It is proved that, with the guidance laws, the line-of-sight angular rate will converge to zero or a small neighborhood of zero before the final time of the guidance process. Furthermore, such guidance laws will ensure finite time convergence and finite time stability in both the planar and three-dimensional environments. Simulation results show that the guidance laws are highly effective.**

## Nomenclature

| | | |
|---|---|---|
| $a_{Mr}$, $a_{M\theta}$, $a_{M\phi}$ | = | missile acceleration along the line-of-sight axes |
| $a_r$, $a_\theta$, $a_\phi$ | = | relative acceleration along line-of-sight axes |
| $a_{Tr}$, $a_{T\theta}$, $a_{T\phi}$ | = | target acceleration along line-of-sight axes |
| $N$ | = | navigation ratio |
| $q$ | = | line-of-sight angle |
| $\dot{q}$ | = | derivative of $q$ with respect to time |
| $\ddot{q}$ | = | second-order derivative of $q$ with respect to time |
| $r$ | = | relative range |
| $\dot{r}$ | = | derivative of $r$ with respect to time |
| $\ddot{r}$ | = | second-order derivative of $r$ with respect to time |
| $t$ | = | time |
| $u$ | = | missile acceleration normal to line of sight |
| $u_r$ | = | missile acceleration along line of sight |
| $V_M$ | = | missile velocity |
| $V_T$ | = | target velocity |
| $w$ | = | target acceleration normal to line of sight |
| $w_r$ | = | target acceleration along line of sight |
| $x_M$, $y_M$, $z_M$ | = | position coordinates of missile in inertial frame |
| $x_T$, $y_T$, $z_T$ | = | position coordinates of target in inertial frame |
| $\theta$ | = | azimuth |
| $\phi$ | = | elevation |
| $\varphi_M$ | = | flight-path angle of missile |
| $\varphi_T$ | = | flight-path angle of target |
| $\psi_M$ | = | heading angle of missile |
| $\psi_T$ | = | heading angle of target |

## I. Introduction

PROPORTIONAL navigation (PN) and its variants have been widely used as homing guidance laws because they are highly efficient and easy for implementation [1–10]. The PN guidance law has the required accuracy to intercept a nonmaneuvering target or a weakly maneuvering target. Further, a missile under a PN guidance law has to have advantages in both maneuverability and agility. However, for the task of intercepting a target with maneuverability close to that of a missile, PN guidance laws are unable to achieve the required precision.

An effective approach to deal with maneuverable targets is to apply a robust guidance scheme. Many existing robust guidance laws, such as $H_\infty$ guidance law [11], $L_2$ gain guidance law [12], Lyapunov-based nonlinear guidance law [13], and first-order sliding-mode guidance laws [14–17] are obtained based on Lyapunov theorems on asymptotic stability or exponential stability. The crucial technique to designing the $H_\infty$ guidance law [11] was to find the analytic solution of the associated Hamilton–Jacobi partial differential inequality of the missile guidance problem. Then, the system describing the missile guidance problem was said to have a $L_2$ gain less than a given level. Because the $L_2$ gain is an index defined in a time horizon from zero to infinity, the $H_\infty$ guidance law is not a guidance law with finite time convergence, although it exhibited strong robustness against disturbances from the target's maneuvers and variations in initial engagement conditions. The $L_2$ gain guidance law [12] was also designed to satisfy the $L_2$ gain and not a guidance law with finite convergent time. In the design of the Lyapunov-based nonlinear guidance law [13], a compact set, into which the state of guidance system converges, was obtained by solving a linear matrix inequalities characterization of the pole placement problem. Certainly, the convergence rate can be adjusted by the pole selection. However, theoretically speaking, for nonmaneuvering targets and targets having a constant acceleration, only asymptotic stability was obtained and demonstrated. Existing first-order sliding-mode guidance laws [14–17] were all designed with Lyapunov theorems on asymptotic stability or exponential stability such that they had not been proved to guarantee a finite time convergence. In short, the theoretical results only indicated that the line-of-sight (LOS) angular rate under the aforementioned guidance laws will converge to zero or a small neighborhood of zero as time approaches infinity. These theoretical findings are inconsistent with practical observations. In many applications, the time of termination is really quite short. For example, in the space interception where a missile is intercepting a ballistic target, sometimes the time of terminal guidance is only several seconds such that the guidance law is required to ensure finite time convergence of the LOS angular rate.

In recent years, the finite time stability for feedback control systems (i.e., the states of the systems converge to their equilibrium point and then stay there) has become an active research area. Finite time control, which is related to finite time stability, was first proposed in [18] in 1986. It has since generated many research activities

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