

# Two-Point Boundary Value Problem Solutions to Spacecraft Formation Flying

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The two-point boundary value problem of a leader–follower spacecraft formation flying in unperturbed elliptical reference orbits is studied. The initial and final relative positions and times and the orbit of the leader are known, and the orbit of the follower must be determined. This problem will be called the relative Lambert's problem. First, we will show this relative Lambert's problem can be solved like the classical Lambert's problem. Then, a set of approximate analytic solutions are obtained through linearizing the Lagrange's time equation. Meanwhile, a simplified Newton–Raphson algorithm is applied to obtain numerical solutions, and the relevant quantities of the leader are used as initial conditions. Here, our special efforts are dedicated to the study of the periodic relative orbits of the follower. In particular, a set of first-order analytic solutions to the relative Lambert's problem are derived from the periodic solutions of Lawden's equations, and a constraint on the leader's true anomalies (implicitly in time) and relative positions is obtained. From that constraint on the radial/in-track plane of the leader local-vertical–local-horizontal frame, we found that, for specified initial and final times, the locus of final positions of the follower with fixed initial position is a straight line, and so is the locus of initial positions with fixed final position. Furthermore, for fixed initial and final positions, the transfer times with either specified initial time or final time can be expressed as the real roots of a cubic equation, for which there are at most three solutions. Several examples will be given to support these conclusions.

## Nomenclature

$a$	= semimajor axis
$c$	= chord
$c_j$	= the $j$ th parameter with respect to orbit element differences
$E$	= eccentric anomaly
$e, \mathbf{e}, \tilde{\mathbf{e}}$	= eccentricity, eccentricity vector, and unit eccentricity vector
$f$	= true anomaly
$i$	= orbit inclination
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	= base vectors of the leader local-vertical–local-horizontal frame
$M$	= mean anomaly
$n$	= mean angular velocity
$p$	= semilatus rectum
$R, \mathbf{R}$	= radius and radius vector with respect to the Earth's center
$\mathbf{r}, r$	= relative position vector and its magnitude
$s$	= semiperimeter of triangle
$t$	= time
$\mathbf{V}$	= velocity vector with respect to the Earth
$\mathbf{v}$	= relative velocity vector observed in the leader local-vertical–local-horizontal frame
$x, y, z$	= follower relative position vector in the leader local-vertical–local-horizontal frame
$\alpha, \beta$	= parameters of Lagrange's time equation
$\Delta$	= difference between the follower and the leader
$\theta$	= orbital transfer angle

$\mu$	= gravity constant of the Earth
$\Omega$	= right ascension of the ascending node
$\omega$	= argument of perigee

## Subscripts

$f$	= follower spacecraft
1, 2	= relative to initial and final positions, respectively

## I. Introduction

COMPARED to single spacecraft, spacecraft formations offer greater flexibility and redundancy at a lower cost. So, much research on relative motion, formation design, and reconfiguration, spacecraft rendezvous, etc., has been conducted during the past decade. The relative motion of the moon with respect to the sun–Earth system was first studied by Hill [1]. Almost one century later, relative motion of spacecraft formation flying, that is, of one spacecraft (follower) following another (leader) in the leader local-vertical–local-horizontal (LVLH) frame, was studied by Clohessy and Wiltshire [2]. This was then extended from circular reference orbits to elliptical reference orbits independently by Lawden [3] and Tschauner and Hempel [4]. Since the 1990s, the literature on spacecraft formation flying has greatly expanded. Inalhan et al. [5] developed an initialization procedure for the solutions of the linearized relative motion equations for elliptical reference orbits. Schaub and Alfriend [6], and Gim and Alfriend [7] considered orbital perturbations such as the Earth's oblateness, which was absent in preceding studies. Carter [8], and Yamanaka and Ankersen [9] obtained a concise and simple state transition matrix, which is important to initial value problems involving the differential equations of relative motion. The two-point boundary value problem (TPBVP) of relative motion, here termed the relative Lambert's problem, where the initial and final relative positions, times, and the orbit of the leader are known, and the orbit of the follower is going to be determined, has not yet been sufficiently studied, though it plays a key role in formation configurations and orbit transfers of spacecraft formations. Deriving from the nonperiodic solution of the homogeneous Clohessy–Wiltshire equations, Mullins [10] obtained a set of solutions to TPBVP in circular reference orbits. In the framework of Keplerian motion, the TPBVP is also known as the classical

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