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Time-Delayed Feedback Control in Astrodynamics

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In this paper we present time-delayed feedback control for the purpose of autonomously driving trajectories of nonlinear systems onto periodic orbits. As the generation of periodic orbits is a major component of many problems in astrodynamics, we propose this method as a useful tool in such applications. To motivate the use of this method we apply it to a number of well-known problems in the astrodynamics literature. First, time-delayed feedback control is applied to control the chaotic attitude motion of an asymmetric satellite in an elliptical orbit. Second, we apply time-delayed feedback control to the problem of maintaining a spacecraft in a periodic orbit about a body with large ellipticity (such as an asteroid). Third, we apply time-delayed feedback control to eliminate the drift between two satellites in low Earth orbits to ensure that their relative motion is bounded.

I. Introduction

TIME-DELAYED feedback control (TDFC) is an efficient method for stabilizing unstable periodic orbits embedded in chaotic attractors. TDFC was first proposed by Pyragas in 1992 [1] as a simple and efficient method to control chaos in systems of ordinary differential equations. TDFC is based on applying a feedback control that is proportional to the deviation of the current state of the system from its state one period in the past: explicitly,

$$\boldsymbol{u}(t) = -K(\boldsymbol{X}(t) - \boldsymbol{X}(t-\tau)) \tag{1}$$

where $\boldsymbol{u}(t)$ is the control, K is a gains matrix, $\boldsymbol{X}(t)$ is the n-dimensional state vector, and $\boldsymbol{X}(t-\tau)$ is the state one period in the past with period τ. The fundamental difference between the control (1) and that of a linear quadratic regulator (LQR) [2] used for practical orbital control is that TDFC tracks a delayed trajectory rather than a reference trajectory. This is fundamentally different, as the delayed trajectory can evolve continuously over time (unlike a prespecified reference trajectory in LQR). The authors propose this method not as an alternative to LQR but as a highly autonomous control (requiring only a prespecified period) that can be used for stabilizing unstable periodic orbits embedded in chaotic attractors and for bounding the motion of unstable trajectories. Furthermore, TDFC has been successful in many applications: for example, in the stabilization of laser coherent modes [3,4], the control of heart conductivity [5], and the control of systems with friction [6]. The main advantages of this method are that is does not require a reference orbit for its implementation and can therefore be applied to systems without a priori knowledge of their dynamics. However, it does require that the period of the orbit be specified a priori, and once this is specified, the target orbit is essentially chosen. To the authors knowledge, the only application of TDFC to a problem in orbital dynamics is in the generation of periodic reference trajectories above the ecliptic plane in the solar sail elliptic three-body problem [7] and in the stabilization of halo orbits [8].

The general problem considered in this paper is to apply TDFC (1) to a nonlinear system of the form

$$\dot{\boldsymbol{X}}(t) = f(\boldsymbol{X}(t)) + B\boldsymbol{u}(t) \tag{2}$$

such that it drives $\boldsymbol{X}(t)$ onto a periodic orbit of some prespecified period τ. Here, B is a constant $n \times m$ matrix, where m is the number of controls, and a dot denotes differentiation with regard to time. Defining the error in the period by the functional

$$\|\boldsymbol{e}(t)\| = \|\boldsymbol{X}(t) - \boldsymbol{X}(t-\tau)\|$$

the general objective is to minimize $\|\boldsymbol{e}(t)\|$ over the time interval $t \in [0, T]$, where T is the final time. If $\|\boldsymbol{e}(t)\| = 0$ at $t = T$, then $\boldsymbol{u}(t) = 0$ and the final state is a natural periodic orbit of the nonlinear system (2).

Traditionally, TDFC has been applied to control chaos in low-dimensional systems of ordinary differential equations. However, it is somewhat surprising that this method has not found its way into mainstream astrodynamic applications, considering the large number of chaotic phenomena that exist [9,10]. This is perhaps due to an incorrect claim that the method has an inherent limitation known as the odd number limitation [11,12]. The odd number limitation asserts that TDFC cannot stabilize an unstable periodic orbit with an odd number of real positive Floquet multipliers greater than 1. However, this limitation has recently been refuted in a number of papers [13–15]. These findings open up the possibility for TDFC to be used in trajectory-tracking applications and to stabilize periodic orbits that are not necessarily embedded in chaotic attractors. To this end, we illustrate how this method has the potential to be used in a variety of applications throughout astrodynamics and practical orbit control. First, we apply TDFC to the problem of controlling the chaotic attitude motion of a satellite in an elliptical orbit. This application falls into the conventional use of TDFC in that the target periodic orbit is embedded in a chaotic attractor.

The subsequent applications in this paper illustrate how TDFC may be used in a new setting that is to drive trajectories onto periodic orbits that are not embedded in chaotic attractors. These novel applications involve using TDFC to bound the motion of unstable trajectories. Previously, TDFC has been applied to stabilize an isolated periodic orbit (i.e., not embedded in a chaotic attractor). Moreover, this was demonstrated through the application of TDFC to stabilizing a halo orbit [8]. However, this application is not considered practically feasible as the instability time scale of halo orbits is much shorter than their orbit period. Thus, even if initially close to a periodic orbit, the instability of the system dictates that the orbit will be quite far from the initial periodic orbit after a full period. In addition, because halo orbits have large periods (around six months), the use of a time lag of that magnitude in a feedback control law would be not be practical, given the diverse issues that can arise over that time span. However, the applications in this paper are deemed practical, as their initial (near-periodic) trajectory has a relatively large instability time scale with much smaller natural periods of several hours. These applications involve using TDFC to 1) drive a spacecraft autonomously onto a periodic orbit of

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