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Engineering Notes

Analysis of Transition Stability for Morphing Aircraft

T. M. Seigler*
University of Kentucky, Lexington, Kentucky 40506-0503
and
D. A. Neal†
Barron Associates, Inc., Charlottesville, Virginia 22901

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I. Introduction

FOR rigid aircraft, there are standard techniques for evaluating flight stability, generally involving the application of linear system theory. The standard approach is to consider a particular flight mode of interest (e.g., steady-wings-level flight, steady turning, steady pull-up, etc.) and evaluate the stability of an equilibrium point in this regime. Because the aerodynamic forces and moments are difficult to express analytically over a large range of possible aircraft motions, most often, equilibrium point stability is evaluated based upon a set of small-perturbation linear equations.

For a morphing aircraft, in addition to fixed planform stability, we are also interested in the transient stage of flight during which the aircraft changes shape. The particular question that we want to address is how the speed of transition affects stability. We refer to this inquiry as transition stability. The term morphing aircraft as it is used to here denotes a flight vehicle that exhibits large, controlled changes in geometry during flight for the purpose of optimal performance in a given flight regime. This definition distinguishes the present considerations from morphing concepts such as wings with conformal control surfaces, which are used as aerodynamic control inputs and do not include a substantial redistribution of mass [1,2].

The assumptions used to evaluate the stability of rigid aircraft are not valid for the morphing aircraft problem. When the mass of the aircraft is substantially redistributed, the moments of inertia and aerodynamic coefficients used for rigid analysis may change significantly. As a result, there will generally be a shift of the equilibrium conditions. Hence, we can no longer speak of asymptotic stability of an equilibrium point, but only of bounded stability. An analytical evaluation of transition stability must be based on a mathematical model of the aircraft. However, as discussed in Sec. II, a detailed dynamic description of complex changes in geometry and the resulting aerodynamic interaction is likely impractical. In subsequent discussion, we attempt to justify two important assumptions of the analysis: the inertial forces due to morphing are negligible, and the aerodynamic forces are dependent solely on the instantaneous configuration of the aircraft. Both assumptions require that the geometry changes occur at a slow rate relative to the flight speed. With these two assumptions, the flight equations can be expressed as a set of parameter-dependent ordinary differential equations, where the parameter is related to the instantaneous shape of the aircraft. The manner in which the parameter is related to the aircraft geometry is not specifically restricted in any manner. Upon linearization about an equilibrium point, the equations become linear parameter varying. Transition stability can then be evaluated by determining the limit on the rate of change of the parameter. For this analysis, the theory of slowly varying systems [3–6] is applicable; the basic theory as it applies to the morphing aircraft problem is outlined in Sec. III. Straightforward application of the theory can be algebraically cumbersome and also requires explicit analytical functions for all parameter-dependent components of the equations of motion. Numerical approximations relieve these difficulties, as demonstrated in Sec. IV with an example.

II. Problem Formulation

In agreement with the traditional notation of flight mechanics, let $v = [U \quad V \quad W]^T$ and $\omega = [P \quad Q \quad R]^T$ quantify the translational and angular velocity of a fixed-body reference frame, which is confined to rotate and translate with some rigid portion of the aircraft [7,8]. Also, let $p = [p_x \quad p_y \quad p_z]^T$ quantify the local position of a material point relative to the fixed-body frame. The matrix form (i.e., reference frame dependent) of the translational and rotational equations of a morphing aircraft can be expressed in the form [9,10] $\mathcal{D}$.

$$m\dot{v} + m\tilde{\omega}v - m\tilde{r}_C\dot{\omega} - \tilde{\omega}m\tilde{r}_C\omega - 2m\dot{\tilde{r}}_C\omega + \ddot{r}_C = F \tag{1}$$

$$m\tilde{r}_C\dot{v} + m\tilde{r}_C\tilde{\omega}v + J\dot{\omega} + \tilde{\omega}J\omega + \dot{J}\omega + \int_{\mathcal{D}} \tilde{p}\,\ddot{p}\;\mathrm{d}m = M \tag{2}$$

where m is the mass, r_C locates the center of mass relative to the fixed-body frame, F and M are the applied force and moment expressed in the body frame, $\mathcal{D}$ is the spatial volume occupied by the body, and the second moment of inertia is

$$J = -\int_V \tilde{p}\,\tilde{p}\;\mathrm{d}m$$

The ($\sim$) notation in Eqs. (1) and (2) denotes the skew symmetric representation of a single-column matrix. The equations reduce to the standard rigid body flight equations when $\dot{p} = 0$ for every material point and the center of mass is chosen as the reference origin, requiring that $r_C = 0$. For the nonrigid case, we can also let the body-fixed axis move with the center of mass, in which case, again, $r_C = 0$. However, v would then be the velocity of the center of mass rather than a fixed point of the aircraft. For sufficiently large aircraft speeds, this difference will have only a small effect. Note also that the moments of inertia may become more complicated when the reference point is in relative motion.

In the wind axis, ignoring the presence of wind gusts and crossflow, etc., the equations of motion become

$$\begin{aligned} &m\dot{v}_w + m(\tilde{\omega}_{w/b} + C_{wb}\tilde{\omega}C_{bw})v_w - mC_{wb}\tilde{r}_C\dot{\omega} - mC_{wb}\tilde{\omega}\tilde{r}_C\omega \\ &\quad - 2mC_{wb}\dot{\tilde{r}}_C\omega + mC_{wb}\ddot{r}_C = C_{wb}F \end{aligned} \tag{3}$$

$$\begin{aligned} &m\tilde{r}_C\dot{v}_w + m\tilde{r}_C(\tilde{\omega}_{w/b} + C_{wb}\tilde{\omega}C_{bw})v_w + C_{wb}J\dot{\omega} + C_{wb}(\tilde{\omega}J + \dot{J})\omega \\ &\quad + C_{wb}\int_V \tilde{p}\,\ddot{p}\;\mathrm{d}V = C_{wb}M \end{aligned} \tag{4}$$

where V_T is the total velocity, and thus the wind-axis velocity is $v_w = [V_T \quad 0 \quad 0]^T$. Also, C_{wb} is an orthonormal operator that transforms vectors expressed in the fixed-body axis into the wind axis, and $\omega_{w/b}$ is the wind-axis representation of the angular velocity

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*Assistant Professor, Department of Mechanical Engineering; seigler@engr.uky.edu. Senior Member AIAA.

†Research Engineer; neal@barron-associates.com. Member AIAA.