Journal of Guidance, Control, and Dynamics
Vol. 32, No. 6, November–December 2009

State Transition Matrix Approximation Using a Generalized Averaging Method

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DOI: 10.2514/1.44142

This paper presents a method for approximating the state transition matrix for orbits around a primary body and subject to arbitrary perturbations. A generalized averaging method is employed to isolate the high- and low-frequency regions of the perturbation terms and to construct a functional form of the approximate state transition matrix composed only of elementary analytic functions. The resulting state transition matrix is expressed with a small number of constant parameter matrices and osculating orbit parameters at an initial epoch and is valid for tens of orbital revolutions without having to update the parameters. Numerical simulations show that this method is valid for arbitrary-eccentricity orbits with semimajor axes ranging from low Earth orbit up to around 10 Earth radii when applied to Earth orbits. This method has been developed for implementation onboard spacecraft for high-accuracy formation-flying missions. Furthermore, it is shown that the symplectic property, which is a fundamental mathematical structure of Hamiltonian systems, can be incorporated into the method. This not only reduces the number of parameters required for approximations, but also preserves the physically true structure of the state transition matrix and provides some important properties that are useful for practical onboard computation.

I. Introduction

A METHOD for approximating state transition matrices for fully perturbed orbital dynamics is described in this paper. The generalized averaging method for linear differential equations with almost-periodic coefficients [1] is applied to derive a simple form for fully perturbed state transition matrices, composed only of elementary analytic functions and a small number of parameter matrices. The primary advantage of this method is that it can provide a simple mathematical form of a fully perturbed state transition matrix that is especially suited to use on onboard systems.

There are numerous studies on state transition matrices for orbital dynamics. The earliest studies on this problem were performed by Hill [2] and Clohessy and Wiltshire [3], who derived the linearized behavior of neighboring orbits of a circular reference orbit assuming two-body dynamics. Lawden [4] developed linearized orbital motion around an eccentric reference orbit, which also assumes two-body dynamics, and its solution form was improved by Carter and Humi [5] and Carter [6].

Most recent developments in theories for the orbital state transition matrix stem from a growing demand for and development of formation-flight technologies. It has been found that even for proximity relative orbital motion, it may be insufficient to only consider the linearized dynamics of Keplerian motions to obtain even moderately accurate relative orbital motions.

Research to overcome this has mainly occurred in three ways. The first direction is to add realistic perturbation effects. For example, the linearized effect of the J2 term can be analytically derived and included into the state transition matrix around circular or eccentric orbits [7–11]. The second direction of study is to take into account not only linearized behavior, but also nonlinear effects. Alfriend et al. [12] gave a second-order relative motion based on Keplerian elements representations. Vaddi et al. [13] presented a state transition matrix that includes the effects of eccentricity, gravitational perturbations and nonlinearities. Melton [14] presented a state transition matrix approach incorporating higher-order eccentricity terms. Park and Scheeres [15] proposed a higher-order Taylor series expansion approach to apply to general (perturbed) trajectories. There are also geometric approaches [16,17] that directly consider the form of solutions for relative orbital motions, rather than linearizing the orbital equations of motion. The third direction is to capture the precise mathematical structure of relative orbital motions. Palmer and Imre [18] derived second-order relative Keplerian motions that precisely conserve the relative Hamiltonian and relative angular momentum. Imre and Palmer [19] presented an integration method of relative orbital motions that exactly conserves the symplectic structure of Hamiltonian system. Tsuda and Scheeres [20] derived a generalized method of solving the symplectic state transition matrix and applied it to fully perturbed relative orbital motions around eccentric Earth orbits and Halo orbits.

On the other hand, many formation-flight missions are proposed or planned with demanding requirements [21]. It is natural to think that larger numbers of satellites with more accurate formation-keeping and control capabilities will be required in the future. For highly autonomous formation control, a precise model of the relative orbital motion must be implemented onboard the spacecraft. This situation is highlighted when a large number of spacecraft are involved, or only infrequent ground support is expected. Thus, for highly accurate autonomous formation flying, not only precise but also simple mathematical representations of relative orbital motions are important.

The aim of this paper is to provide a simple mathematical form for the state transition matrix under a fully perturbed environment. Distinct from the past studies introduced earlier, this method provides a simple functional form of the state transition matrix and provides a method of parameter matching to model state transition matrices for precise relative orbital motions incorporating a variety of practical perturbation sources, such as geopotential perturbations and third-body perturbations. Past studies mainly consider differential geopotential effects as a source of perturbation, but third-body

Received 4 March 2009; revision received 13 June 2009; accepted for publication 13 June 2009.

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