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Engineering Notes

Efficient Initial Costates Estimation for Optimal Spiral Orbit Transfer Trajectories Design

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Introduction

AN INITIAL costates estimation method to solve optimal transfer orbit design problems is presented here. The target orbits are spiral trajectories with low-thrust input. Fuel optimal trajectory design problems have been previously investigated for a number of orbit transfer applications and interplanetary missions [1–7]. Among representative techniques is an indirect approach based on the variational principle, through which the optimal control problems can be converted into a two-point boundary value problem. Transfer orbit design problems can be solved by finding the unknown initial costate variables, but the indirect method suffers from some critical drawbacks, such as a small radius of convergence. The estimation of initial costates for spiral trajectory design is particularly difficult due to a long transfer time and the multirevolution nature of such trajectories. Several advanced indirect methodologies, including functional approximation, continuation, homotopy, and the step-by-step approach, have been adopted to compute initial unknown costates [1,2,8,9].

In [1], properties of the initial costates were exploited to estimate the initial costates. In that work, the terminal specific energy with respect to the Earth and the initial radius of the circular orbit were fixed. The initial costates of the radial distance and tangent velocity can be approximated using exponential functions versus time; however, the initial costate behaviors of radial velocity were not fully analyzed in [1]. Furthermore, if the initial radius, terminal specific energy, and central planet are changed, additional steps, such as determining a new curve fitting, may be necessary to design optimal trajectories with a longer transfer time.

In this study, a new initial guess structure for costates is proposed to construct optimal spiral trajectories using the arbitrary initial radius, transfer time, terminal target energy, and central planet. The proposed initial guess structure requires neither functional approximation nor extrapolation. The structure is developed according to initial costate properties, and it can accommodate specific energy targeting problems.

Problem Definition

In this section, a specific energy targeting problem is investigated. The same system state dynamics are presented in [1,2].

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$$\dot{r} = v_r \tag{1}$$

$$\dot{v}_r = (v_\theta^2/r) - (\mu/r^2) + a_{\text{ref}} u \sin(\alpha) \tag{2}$$

$$\dot{v}_\theta = -(v_\theta v_r/r) + a_{\text{ref}} u \cos(\alpha) \tag{3}$$

$$\dot{\theta} = (v_\theta/r) \tag{4}$$

where a_{ref} represents a reference acceleration magnitude, and u and α are command inputs, with u a normalized acceleration magnitude and α an in-plane thrust angle. The low thrust is assumed to be an unconstrained variable specific impulse (VSI) type of engine, for which the thrust magnitude is modulated by the specific impulse. The mass flow dynamics are prescribed as

$$\dot{m} = -\frac{T}{v_e} = -\frac{2\varepsilon P}{I_{\text{SP}}^2 g_0^2} \tag{5}$$

where I_{sp} denotes specific impulse,g_0 is the Earth's gravitational acceleration at sea level,P is engine power, and ε is engine efficiency. To minimize fuel consumption for an unconstrained VSI engine with a fixed transfer time, the spacecraft mass and power are decoupled from the problem, and only the accumulated thrust acceleration affects the spacecraft propellant [1].

$$\frac{1}{m_f} - \frac{1}{m_0} = \frac{1}{2}\int_{t0}^{tf} \frac{a^2}{P}\,\mathrm{d}t = \frac{a_{\text{ref}}}{2P}\int_{t0}^{tf} u^2\,\mathrm{d}t \tag{6}$$

In Eq. (6), the denominator P is constant at all times, and the numerator term, a_{ref}, to be determined later, is also constant. Therefore, the denominator P and the numerator term, a_{ref}, do not affect the optimal cost. Finally, the cost is adjusted as a square of the normalized thrust magnitude to minimize fuel consumption, such that

$$J = \nu(\varepsilon - \varepsilon_t) - \int_{t_0}^{t_f} u^2\,\mathrm{d}t \tag{7}$$

where $\varepsilon(=v^2/2 - \mu/r)$, and ε_t corresponds to the specific target energy of the spacecraft with respect to the central body.

The Hamiltonian, optimality, necessary, and boundary conditions can be derived from the variational principle. By applying typical optimal control formulation, the following set of equations can be readily derived [1,2]:

$$\dot{\lambda}_r = -\frac{\partial H}{\partial r} = \lambda_{vr}\left(\frac{v_\theta^2}{r^2} - \frac{2\mu}{r^3}\right) - \lambda_{v\theta}\left(\frac{v_r v_\theta}{r^2}\right) + \lambda_\theta\left(\frac{v_\theta}{r^2}\right) \tag{8}$$

$$\dot{\lambda}_{vr} = -\frac{\partial H}{\partial v_r} = -\lambda_r + \lambda_{v\theta}\left(\frac{v_\theta}{r}\right) \tag{9}$$

$$\dot{\lambda}_{v\theta} = -\frac{\partial H}{\partial v_\theta} = -2\lambda_{vr}\left(\frac{v_\theta}{r}\right) + \lambda_{v\theta}\left(\frac{v_r}{r}\right) - \lambda_\theta\left(\frac{1}{r}\right) \tag{10}$$

$$\dot{\lambda}_\theta = -\frac{\partial H}{\partial \theta} = 0 \tag{11}$$

The optimal control input can be derived such that

$$\alpha = \tan^{-1}(\lambda_{vr}/\lambda_{v\theta}) \tag{12}$$