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Minimum-Time Path Planning for Unmanned Aerial Vehicles in Steady Uniform Winds

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This paper is concerned with time-optimal path planning for a constant-speed unmanned aerial vehicle flying at constant altitude in steady uniform winds. The unmanned aerial vehicle is modeled as a particle moving at a constant air-relative speed and with symmetric bounds on turn rate. It is known from the necessary conditions for optimality that extremal paths comprise only straight segments and maximum-rate turns. An essential observation is that maximum-rate turns correspond to trochoidal path segments, as observed from an Earth-fixed inertial frame. The path-planning problem therefore reduces to identifying the switching points at which straight and trochoidal path segments join to form a feasible path and choosing the true minimum-time solution from the resulting set of candidate extremals. The paper's primary contribution is a simple analytical solution for a subset of candidate extremal paths: those for which an initial maximum-rate turn is followed by a straight segment and then a second maximum-rate turn in the same direction as the first. The solution is easy to compute and is suitable for real-time implementation onboard an unmanned aerial vehicle with limited computational power. The remaining candidate extremal paths may be found using a simple numerical root-finding routine. The paper also shows that, for some candidate extremal paths, no corresponding Dubins path exists in the (moving) air-relative frame.

Nomenclature

$\mathcal{F}_A$ = air-relative frame
$\mathcal{F}_I$ = inertial reference frame
$\mathcal{F}_T$ = trochoidal frame
$\mathcal{H}$ = Hamiltonian
$t_{2\pi}$ = time required for one full turn at maximum turn rate
V_a = airspeed of the unmanned aerial vehicle
V_w = wind speed
λ = adjoint variables
χ = course angle
ψ = heading angle
ψ_w = direction of ambient air's motion
ω = turn rate

I. Introduction

THIS paper describes a framework for minimum-time path planning in the horizontal plane. We consider a kinematic model for an unmanned air vehicle (UAV) flying at constant altitude and constant air-relative speed in a steady uniform flowfield. In this setting, we seek the feasible path that brings the UAV from a given initial point and heading to a given final point and heading in the least amount of time. Neglecting the effect of wind, this problem is equivalent to finding the minimum arc-length path of bounded curvature connecting two points in the plane with prescribed initial and final slopes. The problem was formulated and studied by Dubins, who showed, using geometrical considerations, that a minimum-length path contains only maximum-curvature circular arcs and straight segments and, moreover, that it contains three such segments at most [1]. The problem has been reformulated and solved using optimal control theory and Pontryagin's minimum principle in [2]; additional necessary conditions for optimality were provided in [3].

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More recently, the preceding methods have been adopted for time-optimal path planning for UAVs traveling in steady uniform winds [4,5]. In these papers, minimum-time trajectories are designed in the air-relative frame $\mathcal{F}_A$, an inertial frame that moves in the direction of the ambient wind with the same speed. The desired final point in inertial space corresponds to a point in $\mathcal{F}_A$, a virtual target, that moves with the same speed as the wind and in the opposite direction. The challenge is to find the point along the virtual target's path at which a Dubins path intercepts the target. The algorithm iteratively solves the Dubins problem in $\mathcal{F}_A$ until the interception error converges to zero.

Alternatively, one may exploit the geometry of the candidate extremal paths and obtain analytical solutions. The key observation is that a UAV flying in a constant ambient wind with a constant maximum turn rate generates a trochoidal path [6] for which analytical expressions exist. Because extremal trajectories may only contain straight paths and trochoidal segments, one may seek the solution in terms of switching points for which a concatenation of such segments yields a feasible path [7]. Independently of this research, the importance of trochoidal trajectories for minimum-time path planning was recognized in [8]; however, only numerical solutions are presented there.

In this paper, we provide a detailed description of the minimum-time path-planning problem in the plane using trochoidal paths and straight segments. In Sec. II, we set up the problem and introduce the trochoidal frame in which the x axis is aligned with the direction of the ambient air's motion. The trochoidal coordinates expressed in this frame are an essential part of the development. In Sec. III, the general character of extremal paths is discussed. We summarize previous results on minimum-time path planning using Pontryagin's minimum principle and provide an additional necessary condition for optimality. Generalizing Dubins's results, we consider only three-segment extremals, which can be grouped into two major categories, as shown in Fig. 1. Borrowing terminology from [9], the bang–singular–bang (or BSB) extremals are those candidate time-optimal paths for which an initial turn is followed by a straight segment followed by a second turn. The solutions for these paths are presented in Sec. IV. When the initial and final turns have the same sense, these paths can be solved for analytically. The results shed light on the character of the BSB paths in general and suggest an efficient numerical root-finding routine to obtain the remaining candidate BSB trajectories. The bang–bang–bang (or BBB) extremals comprise a turn followed by a second turn in the opposite sense, which is followed by a third turn in the same direction as the first. The solution