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Model Development and Code Verification for Simulation of Electrodynamic Tether System

Joshua R. Ellis* and Christopher D. Hall†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0203

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We develop a numerical model of an electrodynamic tether system composed of two finite end bodies and a flexible tether. The equations of motion of the system are presented along with two methods of discretizing the partial differential equations governing the tether vibrations. The first method is the assumed modes method in which the tether displacements are represented as series of generalized coordinates and assumed mode functions, and the second is a finite element method in which displacements and slopes at points along the tether are interpolated by shape functions. The method of manufactured solutions is used to verify that the computer codes written to simulate the system motion implement the discretization methods properly. Both the assumed modes method and the finite element method perform well for relatively low discretization levels; however, as the number of longitudinal assumed modes is increased, the mass matrix for the longitudinal vibrations becomes poorly conditioned, resulting in numerical errors that can cause the tether vibrations to improperly diverge. This behavior is not seen in the finite element method because the mass matrices for both the transverse and longitudinal vibrations are always well-conditioned.

Nomenclature

$\mathcal{A}$	=	main end body
A_{Mki}	=	assumed-modes-method forcing-vector elements, $\mathrm{m}^2 \cdot \mathrm{s}^{-2}$
$\mathbf{A}_T$	=	acceleration vector used in derivation of tether displacement equations, $\mathrm{m} \cdot \mathrm{s}^{-2}$
A_{Ti}	=	components of $\mathbf{A}_T$ expressed in $\mathcal{F}_E$, $\mathrm{m} \cdot \mathrm{s}^{-2}$
a	=	semimajor axis, m
a_{Di}	=	disturbance acceleration components expressed in $\mathcal{F}_O$, $\mathrm{m} \cdot \mathrm{s}^{-2}$
$\mathbf{B}$	=	central-body magnetic field vector, T
$\mathbf{B}_L$	=	acceleration vector used in determining tether libration equations of motion, $\mathrm{m} \cdot \mathrm{s}^{-2}$
B_{Li}	=	components of $\mathbf{B}_L$ expressed in $\mathcal{F}_E$, $\mathrm{m} \cdot \mathrm{s}^{-2}$
$\mathcal{B}$	=	secondary end body
C_{ij}	=	tether displacement generalized coordinates or degrees of freedom, m
$C_{\Delta h}$, $C_{\Delta t}$	=	constants in discretization error expression
c	=	structural damping constant, s
$\mathrm{d}\bar{s}$	=	differential unstretched tether element
EA	=	longitudinal stiffness of the tether, N
e	=	eccentricity
$\mathbf{e}$	=	set of osculating orbit elements
$\hat{\mathbf{e}}_i$	=	unit axes of $\mathcal{F}_E$
$\mathcal{F}_A$	=	$\mathcal{A}$ body frame
$\mathcal{F}_B$	=	$\mathcal{B}$ body frame
$\mathcal{F}_E$	=	tether-fixed frame
$\mathcal{F}_N$	=	inertial frame
$\mathcal{F}_O$	=	orbital frame
f, F	=	numerical and exact solutions to a set of partial differential equations
$\mathbf{f}_T$, $\mathbf{f}_L$	=	manufactured solution vectors, $\mathrm{m/s^2 m^2} \cdot \mathrm{s}^{-2}$
$\mathbf{f}_i$	=	finite element method external force vector, $\mathrm{m/s^2}$
G_A	=	mass center of $\mathcal{A}$
G_B	=	mass center of $\mathcal{B}$
$\mathbf{g}_i$	=	finite element method gravity vector, $\mathrm{m}^2 \cdot \mathrm{s}^{-2}$
h	=	specific angular momentum, $\mathrm{m}^2 \cdot \mathrm{s}^{-1}$
I	=	inclination, deg
$\mathbf{I}_A$	=	centroidal moment of inertia tensor of $\mathcal{A}$, $\mathrm{kg} \cdot \mathrm{m}^2$
$\mathbf{I}_B$	=	centroidal moment of inertia tensor of $\mathcal{B}$, $\mathrm{kg} \cdot \mathrm{m}^2$
i	=	current in the tether, A
L	=	unstretched length of the tether, m
l	=	orbit parameter, m
ℓ_e	=	length of finite element, m
M_{ij}	=	finite element method mass matrix elements, m
$M_{k,ij}$	=	assumed-modes-method mass matrix elements, m
m_A	=	mass of $\mathcal{A}$, kg
m_B	=	mass of $\mathcal{B}$, kg
N_e	=	number of finite elements
N_i	=	finite element method global shape function
N_i^e	=	finite element method element shape function
N_{ij}	=	tether assumed mode functions
N_L	=	number of longitudinal assumed modes
N_T	=	number of transverse assumed modes
n	=	number of degrees of freedom for each tether displacement in the finite element method
$\hat{\mathbf{n}}_i$	=	unit axes of $\mathcal{F}_N$
O	=	inertial origin
$\hat{\mathbf{o}}_i$	=	unit axes of $\mathcal{F}_O$
P_A	=	tether attachment point on $\mathcal{A}$
P_B	=	tether attachment point on $\mathcal{B}$
p	=	spatial order of accuracy
$\mathbf{p}_A$	=	tether attachment vector on $\mathcal{A}$, m
$\mathbf{p}_B$	=	tether attachment vector on $\mathcal{B}$, m
q	=	temporal order of accuracy
$\mathbf{R}_T(\bar{s}, t)$	=	position vector of $\mathrm{d}\bar{s}$ relative to O, m
r	=	discretization refinement factor
$\mathbf{r}$	=	position vector of $\mathrm{d}\bar{s}$ relative to P_A
r_A	=	orbital radius of G_A, m
$\mathbf{r}_A$	=	position vector of G_A, m
$\mathbf{r}_B$	=	position vector of G_B, m
r_L	=	distance between P_B and P_A, m
$\mathbf{r}_L$	=	position vector of P_B relative to P_A, m
$\bar{s}$	=	tether unstretched arc length, m
$\bar{s}_0$, $\bar{s}_f$	=	value of $\bar{s}$ at the beginning an end of a finite element, m

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*Graduate Student, Aerospace and Ocean Engineering Department; joellis3@vt.edu.
†Professor, Aerospace and Ocean Engineering Department.