JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 32, No. 6, November–December 2009

Engineering Notes

Simplified Singularity Avoidance Using Variable-Speed Control Moment Gyroscope Null Motion

Jay McMahon* and Hanspeter Schaub†
University of Colorado at Boulder, Boulder, Colorado 80309-0431

DOI: 10.2514/1.45433

I. Introduction

SINGULARITY avoidance in variable-speed control moment gyroscope (VSCMG) systems can require significant computation to determine null motion steering commands. This paper presents a less complicated method of determining appropriate null motion steering laws while achieving similar performance with current, more complicated, methods.

Control moment gyroscope (CMG) clusters, which are often used for spacecraft attitude control, can encounter singular gimbal-angle configurations for which a general three-dimensional torque cannot be produced. Such singularities can be overcome through a variety of methods, such as those presented in [1,2]. Another option to help avoid singularities is to use VSCMG devices, which allow a CMG device to vary its wheel speed, and thus produce a torque of about two orthogonal axes (wheel spin and transfer axis) [3–5]. A VSCMG cluster will not encounter gimbal locks (singular gimbal-angle configurations) due to the reaction wheel (RW) modes. If all the CMG torque axes lie in a plane, then one of the RW control axes will point apart from this CMG torque plane. Thus, a VSCMG cluster can always produce the required torque of a chosen attitude control law without encountering temporary small attitude errors, as the CMG singularity is avoided. However, using RW modes to drive through the CMG singular configuration requires significant RW motor torques, which is not power effective [6]. Using the null space of the VSCMG system wisely can enable the system to avoid this singular CMG situation while completing the required control maneuvers. The extra RW control modes of the VSCMG allow for greater effectiveness to rearrange the gimbal angles away from the CMG singularity [7]. This increased null space of the VSCMG devices can also be used to create novel combined attitude and energy storage devices [8–10]. Here, the rotor speed can be spun up during sunlit portions of the orbit to store energy without changing the spacecraft attitude. Then, during a shaded orbit region, the rotors can be spun down using the VSCMG null space to extract this energy again.

VSCMG steering laws lead to a simple condition that maps the desired rotor accelerations and gimbal rates into the required attitude control torque. The null space of this mapping is exploited by Schaub and Junkins using a gradient-based method to drive the gimbal angles away from a CMG singularity [7]. The condition number of the mapping matrix is used as the singularity index. Yoon and Tsiotras analyzed the CMG singularities of VSCMG devices in [11] and provided a small modification to the gradient-based null space proposed in [7]. The advantage of this modification is that stability of the singularity avoidance can be analytically guaranteed. However, this new null space steering law requires particular control of both the wheel speed and the gimbal rate, whereas the earlier method only required gimbal-angle motion. The reduced actuation requirements are a benefit because this makes it easier for the null space to be used to implement auxiliary objects, such as power storage demands or returning the wheel speeds to their original values and avoiding long-term rotor speed drift. Lee et al. presented, in [12], a general formulation to develop optimal null space VSCMG steering laws to avoid a CMG singularity. Their method can account for higher-order CMG cost function sensitivities and provide analytical stability guarantees. However, as with the method by Yoon and Tsiotras, the VSCMG steering law dictates both rotor speed and gimbal-angle changes. If reduced to a simple first-order form, their general formulation can be shown to be a generalization of the earlier methods discussed in [7,11].

Many VSCMG null space steering methods have developed their formulation around the attitude regulation problem. The algebraic null space formulation often becomes significantly more complex if an attitude tracking problem is considered. This Technical Note investigates a simplified CMG singularity measure for which the performance is equivalent to the previously published methods, but it is implemented using a substantial reduction in complexity. In particular, considering an attitude tracking problem does not lead to an increase in complexity. The developments are performed, and numerically simulated, using the optimal steering formulation by Lee et al. [12]. However, the presented results could also be easily applied to the VSCMG null space steering law presented by Yoon and Tsiotras [11].

II. Steering Law Overview

Figure 1 illustrates the gimbal frame coordinate system $\mathcal{G}$: $\{\hat{\boldsymbol{g}}_s, \hat{\boldsymbol{g}}_t, \hat{\boldsymbol{g}}_g\}$ used to describe the time-varying orientation of the VSCMG relative to the spacecraft body $\mathcal{B}$. The gimbal rate $\dot{\gamma}_i$ is applied about the body-fixed axis $\hat{\boldsymbol{g}}_{g_i}$, whereas the rotor speed motor causes angular accelerations $\dot{\Omega}_i$ about the spin axis $\hat{\boldsymbol{g}}_{s_i}$. All VSCMG steering laws for both attitude regulation and tracking application lead to a control condition of the form [3–5,13]:

$$[Q]\dot{\boldsymbol{\eta}} = \mathbf{L}_r \tag{1}$$

$$[Q] = [D_0 \quad D] \tag{2}$$

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \dot{\boldsymbol{\Omega}} \\ \dot{\gamma} \end{bmatrix} \tag{3}$$

Here, $[D_0]$ and $[D]$ are $3 \times N$ matrices, with N being the number of VSCMGs in the system. The projection matrix $[Q]$ is therefore a $3 \times 2N$ matrix that maps the VSCMG control states to the required $N \times 1$ control vector $\mathbf{L}_r$. This Technical Note follows the notation setup in [5,7], which also provide expressions for $\mathbf{L}_r$ for attitude regulation and tracking control formulations. Finally, the parameters $\dot{\boldsymbol{\Omega}}$ and $\dot{\boldsymbol{\gamma}}$ are the $N \times 1$ wheel speed and gimbal rate vectors, respectively.

The wheel speed rate control matrix $[D_0]$ is formulated by [5]

$$[D_0] = [\cdots \quad \hat{\mathbf{g}}_{s_i} \quad J_{s_i} \quad \cdots] \tag{4}$$

Received 13 May 2009; revision received 3 August 2009; accepted for publication 3 August 2009.

*Graduate Research Assistant, Aerospace Engineering Sciences, 431 UCB.

†Associate Professor, H. Joseph Smead Fellow, Aerospace Engineering Sciences Department, 431 UCB. Associate Fellow Member AIAA.