JOURNAL OF PROPULSION AND POWER
Vol. 25, No. 6, November–December 2009

Methodology and Historical Perspective of a Hall Thruster Efficiency Analysis

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DOI: 10.2514/1.38092

A Hall thruster performance architecture was developed based on separation of the total thrust directed along thruster centerline into mass-weighted and momentum-weighted terms. With this formulation, the total thruster efficiency equation was analytically decomposed to explicitly account for the effects of energy conversion losses, plume divergence, and the velocity distribution function of the propellant jet. Thruster efficiency is defined as the product of 1) energy efficiency, 2) propellant efficiency, and 3) beam efficiency. Energy efficiency comprises losses due to ionization processes and losses that manifest as Joule heating, and contains no information about the vector properties of the jet. Propellant efficiency incorporates losses from dispersion in the jet composition and is unity for 100% ionization to a single ion species. The effect of neutrals on dispersion of the jet velocity distribution function in propellant efficiency is introduced in the neutral-gain utilization. The beam efficiency accounts for divergence of the jet and is ideal when the ion velocity vectors are parallel to the thrust axis. Plume divergence is defined as a momentum-weighted term, and the approximation as a charge-weighted term is characterized. The efficiency architecture is derived from first principles and is applicable to all propulsion employing electrostatic acceleration, including Hall thrusters and ion thrusters. Distinctions and similarities to several past methodologies are discussed, including past ion thruster analyses, early Russian performance studies, and contemporary architectures. To illustrate the potential for enhanced understanding of loss mechanisms and ionization processes with an array of far-field plume diagnostics, a case study is presented of low-discharge voltage operation from a 6 kW laboratory Hall thruster.

Nomenclature

A = spherical shell surface area element in the plume
E_1 = voltage exchange parameter
E_2 = mass exchange parameter
$\mathbf{F}$ = thrust density vector in the plume
$\mathcal{F}$ = Faraday constant, $96,485\ \mathrm{C/mol}$ of charge
f_j^* = normalized ion mass flow fraction of jth ion species
$f(\mathbf{v})$ = velocity distribution function of ions and neutrals
g = Earth's gravitational constant at sea level, $9.806\ \mathrm{m/s^2}$
I_{Axial} = axial component of beam current parallel to thruster centerline
I_{Beam} = integrated beam current
I_d = anode discharge current
I_{sp} = total specific impulse
$J(\theta)$ = current density in the plume at angular position θ
j = propellant charge state index, 0, 1, 2, 3, etc., for Xe^0, Xe^{+1}, Xe^{+2}, Xe^{+3}
$\mathcal{M}$ = molecular weight of propellant, $\mathrm{Xe} = 0.1313\ \mathrm{kg/mol}$
$\dot{m}_i$ = mass flow rate of ions, where $\sum \dot{m}_j = \dot{m}_i$ for $j = 1$ to the jth ion species
$\dot{m}_j$ = mass flow rate of jth species
$\dot{m}_T$ = total mass flow rate to the anode and cathode, where $\sum \dot{m}_j = \dot{m}_T$ for $j = 0$ to the jth ion species
$\dot{m}(\theta)$ = mass flux at angular position θ
P_d = discharge power to the anode
P_{jet} = jet power
P_{loss} = power lost to Joule heating processes
P_{min} = minimum power required to sustain ionization
Q = average charge of propellant ions
R = downstream measurement radius from the axis of rotation
r = fraction of electron current to the anode, electron recycle fraction
T = component of thrust vector directed along thruster centerline
V_a = most probable ion acceleration voltage
V_d = anode discharge voltage
$\bar{\mathbf{v}}$ = average exit velocity of the velocity distribution function over velocity space $d\mathbf{v}$ at angular position θ
$\bar{v}$, $\overline{v^2}$ = average propellant velocity, squared propellant velocity
$\bar{v}_i$, $\overline{v_i^2}$ = average ion velocity, squared ion velocity
v_j = exit speed of jth species
$\bar{v}(\theta)$ = radial component of $\bar{\mathbf{v}}$ in hemispherical coordinates from thruster centerline at angular position θ
y_j = normalized speed ratio of the jth species $= v_j/|v_1|$
Z_j = ion charge state $= 1, 2, 3$ for Xe^{+1}, Xe^{+2}, Xe^{+3}
β = fractional loss of acceleration potential
δV_j = acceleration potential of jth ion species
ε_B = average ionization cost per beam ion
$\varepsilon_{B,\mathrm{min}}$ = minimum ionization cost per beam ion
ε_j = ionization potential of jth ion species (12, 33, 65 eV from neutral ground state for Xe)

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