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Resonant Response of Mistuned Bladed Disks Including Aerodynamic Damping Effects

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A mathematical model is developed to investigate the effects of aerodynamic damping on the maximum-amplification factor of mistuned bladed disks. LINSUB, an inviscid linearized unsteady aerodynamic damping code, provides aerodynamic damping influence coefficients that are incorporated into a partial mistuning model that takes advantage of mode localization. This mistuning analysis is then used to demonstrate the effects of aerodynamic damping on the maximum-amplification factor of mistuned bladed disks. The relative importance of aerodynamic effects is determined by a comparison of aerodynamic and structural damping factors. It is shown that neglecting unsteady aerodynamics may result in the predicted optimal mistuning pattern not being optimum in the actual operating environment wherein unsteady aerodynamic effects are present.

Nomenclature

$C(\beta_r)$ = unsteady aerodynamic coefficient
$\widehat{C}_j^k$ = aerodynamic influence of airfoil, motion of k on airfoil j
c = airfoil chord length
$\underline{h}^k$ = tuned response to single airfoil forcing on airfoil k
k = reduced frequency, $\omega c/U$
k_c = structural coupling stiffness
k_j = structural stiffness of airfoil j
k_t = mean or tuned structural stiffness
L_j = generalized aerodynamic force of airfoil j
m = generalized inertia per unit span
N = number of airfoils in row
ND = nodal diameter
r = engine order of excitation
s = number of airfoils affected by mistuning
u = fluid velocity
X = generalized displacement
β_r = interblade phase angle, $2\pi r/N$
Γ_j^k = dimensional aerodynamic influence coefficient
δk_j = mistuned stiffness of airfoil j
ζ^{aero} = dimensionless aerodynamic damping factor
ζ^{mech} = dimensionless mechanical damping factor
ρ = fluid density
ω_r = resonant frequency of airfoil row

Superscripts

d = directly affected by mistuning
i = indirectly affected by mistuning
t = tuned

I. Introduction

HIGH-CYCLE fatigue (HCF) of turbomachine blading resulting from flow-induced vibration is a significant problem throughout the gas turbine industry. To address this problem, various approaches have been developed to predict airfoil resonant response. In these, the response of a tuned airfoil row, that is, a rotor with all airfoils having the same structural properties and thus identical natural frequencies, is analyzed.

In fact, there are small airfoil-to-airfoil structural property variations that result, for example, from the manufacturing process or as a consequence of in-service wear. These are collectively referred to as mistuning and are known to lead to significant increases in airfoil resonant response amplitude as compared with that of the tuned airfoil row, with mistuning thus often cited as an HCF source. Hence, the key metric that characterizes the resonant response of mistuned bladed disks is the amplification factor, the ratio of the largest response amplitude of a mistuned bladed disk to that of a tuned bladed disk.

The earliest mistuning analyses were deterministic and used simplified models to produce closed form expressions to bound the mistuned rotor maximum response [1,2]. More recently, Kenyon and Griffin developed a more general analysis that recovers earlier closed form expressions as special cases [3]. Direct numerical optimization to estimate the maximal response over a set of mistuning variations is an alternative deterministic approach [4,5]. Of particular interest herein is the mistuning model developed by Rivas-Guerra and Mignolet that takes advantage of mode localization by only considering a few airfoils on the rotor, that is, airfoils far from the maximum-amplitude airfoil can be considered as tuned airfoils [6]. This partial mistuning model results in a reduction in the number of variables in the optimization procedure to determine the mistuning pattern that results in the largest amplification factor.

One significant phenomenon not addressed in these mistuning models is the airfoil row unsteady aerodynamics. Because damping is known to be the important parameter controlling maximum-resonant-response amplitude, it might be expected that the unsteady aerodynamics resulting from the vibration of the blading itself, specifically the aerodynamic damping, will have a significant effect on the resonant response amplitude of tuned and mistuned bladed disks. Note that the mechanical damping is considerably reduced in newer rotor designs, particularly those with integral bladed rotors (IBRs) and those without shrouds. As a result, it is anticipated that aerodynamic damping will be particularly important in the vibratory stress analysis of IBRs. Specifically, IBRs inherently have very low mechanical damping as compared with a traditional bladed rotor. This very low mechanical damping of IBRs implies that they are highly susceptible to HCF and also that aerodynamic damping is a

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