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Technical Notes

Singular Spectrum Analysis Applied to Time-Series Measurements in a Self-Excited Tube Combustor

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Introduction

PRACTICAL propulsion systems (such as rocket engines and thrust augmenters) are often plagued by thermoacoustic instabilities (TAIs) arising from the coupling of chamber acoustics and combustion heat release. This coupling is typically represented as a feedback loop established when oscillations in acoustic pressure and heat release occur in phase with each other, that is, when the Rayleigh criterion is satisfied [1]. In general, thermoacoustic excitation relies on locating the heat source such that the oscillations in particle velocity u' lead those of acoustic pressure p' by $\sim$90 deg, producing a positive Rayleigh index Ra [2].

To sustain a TAI, the instability must be driven to overcome the per-cycle acoustic losses of the system. This condition assumes that sound radiation and boundary-layer losses make up the bulk of acoustic losses [3]. TAI growth is typically simplified to be exponential in nature [1], even though its amplitude cannot physically increase indefinitely. As the amplitude of a TAI grows, nonlinear effects involving acoustics and heat release become increasingly important [4]. A constant amplitude regime is then reached, known as the limit cycle.

Thermoacoustic instabilities almost always rely on excitation of resonant modes of the combustion chamber. Practical combustors have many complex modes, which are typically excited simultaneously and are extremely sensitive to both initial and boundary conditions [5]. An in-depth review of combustion instabilities in practical combustion systems can be found in an edited volume by Lieuwen and Yang [6]. It is necessary to identify a simple system for which various parameters governing the onset and sustenance of combustion instabilities can be isolated and studied independently. In this context, one such device which produces TAIs is a resonant tube [7]. The most common resonant tube is the Rijke tube, a pipe that is acoustically open on both ends. For the measurements reported in this study, we employed a Schmidt tube, a resonant tube with a closed–open acoustic configuration.

Although resonant-tube combustors provide ample opportunity for examination of fundamental thermoacoustic processes, the research community is still limited by the range of available diagnostic techniques. Detection systems capable of measuring heat release parameters with high spatial and temporal resolution are scarce. Radical species such as hydroxyl (OH) and methylidene (CH) have been extensively used as approximate markers of combustion heat release [8,9]. Laser-induced fluorescence (LIF) measurements with high spatial resolution have been applied to detect OH and CH in combustion systems [10]. However, most experimental investigations employing LIF measurements use laser systems with repetition rates several orders of magnitude lower than the fundamental frequencies of TAIs. Picosecond time-resolved laser-induced fluorescence (PITLIF) was developed to facilitate time-series measurements of OH concentrations in combustion systems [11]. This technique can facilitate a wide range of temporally and spatially resolved OH measurements at data acquisition rates necessary to resolve transient thermoacoustic processes.

In this study, we employ the PITLIF technique to measure OH time series and a dynamic pressure sensor to obtain pressure time series in a Schmidt tube combustor. The stationary (limit cycle) and nonstationary (transient) portions of simultaneous hydroxyl and pressure time series are analyzed by employing singular spectrum analysis (SSA). The noise-reduction capabilities of SSA are discussed from the point of view of instability mitigation in propulsion systems.

Singular Spectrum Analysis

Statistical discrimination of nonstationary processes is important in the study of practical propulsion systems [12]. Traditionally, signals embedded in a noisy series have been analyzed by statistical pattern-recognition techniques. The extension of classical pattern-recognition techniques to experimental time series has been a problem of great practical interest. In many practical problems, the time series are actually realizations of complex nonstationary processes. Various models of complex nonstationary processes have been proposed in the literature.

In this context, an important analytical procedure used for signal-to-noise enhancement, data compression, and pattern recognition for any time series is singular spectrum analysis [13]. Traditionally used for short, noisy time series, SSA helps to separate the time series into components that can be classified as trends, oscillations, and noise. An important feature of SSA is that the underlying oscillations can be phase- and/or amplitude-modulated; moreover, the trends can be nonlinear. In the current study, we use SSA to extract local features of both stationary and nonstationary time series with a strong focus on combustion instability.

The starting point of SSA is to embed a time series $[X(t){:}t=1,\ldots,N]$ into a vector space of dimension $M < N$. This approach essentially separates the time series into lagged copies through a set of overlapping M-point "windows." These lagged copies are then used to create a new series $X^*(t)$ of M-dimensional vectors so that

$$X^*(t) = (X(t), X(t+1), \ldots, X(t+M-1)) \qquad (1)$$

The vectors in $X^*(t)$ are referenced by $t = 1, \ldots, N'$, where $N' = N - M + 1$. The selection of M depends on balancing the amount of information captured within each window (achieved by choosing a large value for M) with some level of statistical confidence in that information (which requires a large N/M ratio).

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