

Technical Notes

Simplified Theory for Ignition Times of Hypergolic Gelled Propellants

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Introduction

A LONG with current studies of the combustion of gelled fuels [1–3] is interest in hypergolic propellant combinations [4,5]. Complementary to detailed numerical computations of flow and combustion-chamber processes [5–8] are simplified models that can be used for making quick and inexpensive estimates of ignition times subsequent to the injection of cold hypergolic propellants. Since short ignition times can produce unacceptably high pressure peaks in the combustion chamber [8], considerations of adjustment of parameters appearing in ignition-time formulas derived from simplified models may suggest approaches to the achievement of acceptable performance. The objective of the present contribution is to provide one such formula from basic theory, identifying relevant parameters that deserve further study.

Simplified Model

Consider two adjacent half-spaces, one occupied by fuel and the other by oxidizer. The constant thermal conductivity, heat capacity, and density of each may differ, so that the thermal diffusivities α_F and α_O , where the subscripts F and O identify fuel and oxidizer, respectively, need not be the same. If x denotes the spatial coordinate normal to the planar interface and t time, then the heat equation obeyed by the temperature T of each propellant is

$$\alpha_i \partial T / \partial t = \partial^2 T / \partial x^2, \quad i = F, O \quad (1)$$

Let λ_i denote the thermal conductivity of propellant i , and presume a hypergolic exothermic Arrhenius reaction at the interface with overall activation energy E and constant prefactor Q , energy released per unit area per unit time. This can be related to the product of the heat of combustion per unit mass of reactants consumed and the mass rate of consumption of reactants per unit volume by multiplying by a characteristic thickness of the interface. The statement of energy conservation across the interface, having temperature T_I , conveniently placed at $x = 0$, can then be written as

$$\lambda_F \partial T_F / \partial x + \lambda_O \partial T_O / \partial x = -Q e^{-E/RT_I} \quad \text{at } x = 0 \quad (2)$$

where R is the universal gas constant and, for convenience, x is defined to be positive in each propellant.

The problem addressed is one in which the fuel at temperature T_{F_o} and oxidizer at temperature T_{O_o} are brought together instantaneously at time $t = 0$, as may happen, for example, in an impinging-stream

combustor at the moment of propellant injection. The initial and boundary conditions that complete the specification of a well-posed problem are then

$$\begin{aligned} T_F &= T_{F_o} & \text{at } t = 0 (0 < x < \infty) & \quad \text{and} & \quad \text{at } x = \infty (0 < t < \infty) \\ T_O &= T_{O_o} & \text{at } t = 0 (0 < x < \infty) & \quad \text{and} & \quad \text{at } x = \infty (0 < t < \infty) \end{aligned} \quad (3)$$

Equation (1) is to be solved in the region $0 < t < \infty$ and $0 < x < \infty$, subject to the boundary condition at $x = 0$ in Eq. (2) for $0 < t < \infty$ and to the initial and boundary conditions in Eq. (3).

Derivation of the Integral Equation

The first task is to determine the interface temperature T_I at $t = 0$: namely, T_{I_o} . This is straightforward because the finite rate exothermic reaction has not yet begun. From the well-known error-function solution

$$T_i = T_I + (T_{i_o} - T_I) \operatorname{erf}[x / (2\sqrt{\alpha_i t})], \quad i = F, O \quad (4)$$

resulting in

$$\lambda_i (\partial T_i / \partial x)_{x=0} = \lambda_i (T_{i_o} - T_I) / \sqrt{\pi \alpha_i t}, \quad i = F, O \quad (5)$$

it is seen by application of Eq. (2) with zero on the right-hand side that

$$T_{I_o} = (r_F T_{F_o} + r_O T_{O_o}) / (r_F + r_O) \quad (6)$$

where $r_i = \lambda_i / \sqrt{\alpha_i}$ could be called the thermal responsivity of propellant i . Given this initial interface temperature, the principal interest then is to find how T_I varies with time.

Since the temperature differences $T_i - T_{I_o}$ have null initial and boundary conditions, the Laplace transforms

$$\bar{T}_i = \int_0^\infty (T_i - T_{i_o}) e^{-st} dt, \quad i = F, O \quad (7)$$

which are found from Eq. (1) to obey second-order linear ordinary differential equations in x , must have the bounded solutions

$$\bar{T}_i = \bar{F}_i e^{-x\sqrt{s/\alpha_i}}, \quad i = F, O \quad (8)$$

where the functions $\bar{F}_i(s)$ are to be determined by the boundary conditions at $x = 0$. Continuity of temperature at $x = 0$ implies that

$$\bar{F}_F - \bar{F}_O = (T_{O_o} - T_{F_o}) / s \quad (9)$$

and the Laplace transform of Eq. (2) is, in view of Eq. (8),

$$\sqrt{s}(r_F \bar{F}_F + r_O \bar{F}_O) = Q \int_0^\infty e^{-E/RT_I} e^{-st} dt \quad (10)$$

In terms of the transform

$$\bar{F} = (r_F \bar{F}_F + r_O \bar{F}_O) / (r_F + r_O) \quad (11)$$

Eq. (9) implies that

$$\begin{aligned} \bar{F}_F &= \bar{F} + r_O (T_{O_o} - T_{F_o}) / [(r_F + r_O)s] \\ \bar{F}_O &= \bar{F} + r_F (T_{F_o} - T_{O_o}) / [(r_F + r_O)s] \end{aligned} \quad (12)$$

and Eq. (10) is

$$\sqrt{s}(r_F + r_O) \bar{F} = Q \int_0^\infty e^{-E/RT_I} e^{-st} dt \quad (13)$$

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