

Efficient Calculation of Radiation Heat Transfer in Participating Media

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A low-cost computational solution to radiation problems can be obtained by using a simple model, such as the P_1 model, but the accuracy can be very poor. High accuracy can be obtained by solving the radiative transfer equation, but the solution cost can be exorbitant for strongly participating media. The Q_L method presented in this paper allows the radiation heat transfer to be computed from a single equation for the average intensity, like the P_1 model, but the Q_L equation contains parameters that account for a nonuniform intensity distribution. The method converges to the solution of the radiative transfer equation with grid refinement and will accommodate any scattering phase function. For a given spatial and directional discretization, and for problems involving radiation only, the accuracy of the Q_L method is shown to equal or exceed that of the finite volume method. The solution cost of the Q_L method is comparable to the finite volume method for weakly participating media, but for strongly participating media the Q_L method is much less costly. The Q_L method is designed for application in general-purpose codes in which radiation is but one of several important processes, and it is in such applications that the major benefits of the Q_L method are expected.

Nomenclature

AR	= aspect ratio
a	= coefficient in the algebraic equations
a_1	= coefficient in the linear scattering phase function
B_x, B_y	= size of the coarse-mesh block in the multigrid solver
b	= source term in the algebraic equations
D_{ij}^l	= defined in Eq. (23), sr
\mathbf{e}_i	= unit vector in the i direction
F_i^l	= defined in Eq. (23), sr
I	= radiant intensity, $\text{W}/\text{m}^2 \cdot \text{sr}$
I_a	= average intensity, $\text{W}/\text{m}^2 \cdot \text{sr}$
I_b	= blackbody intensity, $\text{W}/\text{m}^2 \cdot \text{sr}$
\mathbf{I}_s	= scattering integral, W/m^2
\mathbf{I}_t	= transport integral, W/m^3
I_P^l	= intensity at node P within ω^l , $\text{W}/\text{m}^2 \cdot \text{sr}$
K	= absorption coefficient, m^{-1}
L	= total number of discrete solid angles
L_x, L_y	= dimensions of a rectangular enclosure, m
N_x, N_y	= number of control volumes in the x and y directions
N_θ, N_ϕ	= number of discrete solid angles in the θ and ϕ directions
N_i^l	= defined in Eq. (29), sr

\mathbf{n}	= unit surface normal, out of medium
q	= net surface heat flux, W/m^2
\mathbf{q}	= radiant heat flux vector [Eq. (9)], W/m^2
q_R	= incident radiant heat flux onto a surface, W/m^2
R	= residual in each cycle [Eqs. (31) and (34)]
Rng	= range of intensity in the domain
r	= residual in each multigrid iteration [Eq. (32)]
\mathbf{r}	= spatial position vector, m
S	= surface area, m^2
S_i	= defined in Eq. (25), sr
s	= distance along a beam in the \mathbf{s} direction, m
\mathbf{s}	= unit vector in the direction of intensity
s_i	= component of \mathbf{s} in direction \mathbf{e}_i
\mathbf{s}^l	= unit direction vector in the center of discrete solid angle ω^l
T	= temperature, K
T_G	= temperature of the medium, K
T_h, T_c	= temperature of the hot and cold walls, K
T_{ij}	= defined in Eq. (25), sr
T_{\max}, T_{\min}	= maximum and minimum wall temperatures, K
V	= volume, m^3
x, y, z	= Cartesian coordinates, m
x_i	= distance in the \mathbf{e}_i direction, m
α^l	= phase weight [Eq. (20)]
Δ	= width of the interior control volumes in a uniform grid, m
ϵ	= emissivity
θ	= polar angle measured from the z axis, rad
κ	= extinction coefficient ($K + \sigma^s$), m^{-1}
κ^*	= optical thickness (κL_y)
ρ	= reflectivity
σ	= Stefan–Boltzmann constant ($= 5.729 \times 10^{-8} \text{ W}/\text{m}^2 \cdot \text{K}^4$)
σ^s	= scattering coefficient, m^{-1}
$\Phi(\mathbf{s}', \mathbf{s})$	= phase function for scattering from \mathbf{s}' to \mathbf{s}
$\bar{\Phi}^l$	= defined in Eq. (5), sr
ϕ	= azimuthal angle in the x – y plane measured from the x axis, rad
Ψ	= scattering angle between incident (\mathbf{s}') and scattered (\mathbf{s}) directions, rad

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