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Free Convection About a Vertical Circular Plate

S. C. Haldar*
University of the West Indies at St. Augustine, Trinidad, Trinidad and Tobago, West Indies

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This paper reports on a study of a three-dimensional numerical solution of free convection of air about a vertical circular plate for isothermal and isoflux heating conditions. The governing equations were solved in a vorticity and vector potential formulation. Fluid is drawn toward the heated plate from the bottom and the front of it. The fluid converges as it approaches the plate and moves almost vertically upward close to the plate. For the case of isoflux heating, the plate temperature at a particular radius increases with height and the location of maximum temperature shifts upward with increasing Grashof number. For both isothermal and isoflux heating conditions, the correlations between the Nusselt and Grashof numbers for the case of the circular plate have been developed as a function of the same for the flat plate under similar conditions.

Nomenclature

g^* = acceleration due to gravity, m s^{-2}
Gr = Grashof number, $g^*\beta^*T^*_{\mathrm{ref}}L^{*3}_{\mathrm{ref}}/\nu^{*2}$
k^* = thermal conductivity, W · m^{-1} · K^{-1}
L^* = height of flat plate, m, 1
L^*_{ref} = reference length, m, L^* for the flat plate and r^*_p for the circular plate
Nu = average Nusselt number, $h^*L^*_{\mathrm{ref}}/k^*$
Pr = Prandtl number
q^* = heat flux, W · m^{-2}
r^* = radial coordinate, m, Fig. 1, r^*/r^*_P
r^*_P = plate radius, m, 1.0
Ra = Rayleigh number, $Gr \times Pr$
T^* = temperature, K, $(T^* - T^*_\infty)/T^*_{\mathrm{ref}}$
T^*_{ref} = reference temperature, K, $(T^*_P - T^*_\infty)$ for the isothermal plate and $q^*L^*_{\mathrm{ref}}/k^*$ for the isoflux plate
T^*_∞ = freestream temperature, K, 0
$u*$ = velocity along r^*, m · s^{-1}, $u^*/(\nu^*/r^*_P)$
v^* = velocity along θ, m · s^{-1}, $v^*/(\nu^*/r^*_P)$
w^* = velocity along z^*, m · s^{-1}, $w^*/(\nu^*/r^*_P)$
z^* = axial coordinate, m, Fig. 1, z^*/r^*_P
β^* = coefficient of volumetric expansion, K^{-1}
θ = angular coordinate, Fig. 1
ν^* = kinematic viscosity, m^2 · s^{-1}
Ψ^*_r = r^* component of vector potential, m^2 · s^{-1}, Ψ^*_r/ν^*
Ψ^*_θ = θ component of vector potential, m^2 · s^{-1}, Ψ^*_θ/ν^*
Ψ^*_z = z^* component of vector potential, m^2 · s^{-1}, Ψ^*_z/ν^*
Ω^*_r = r^* component of vorticity, s^{-1}, $\Omega^*_r/(\nu^*/r^{*2}_P)$
Ω^*_θ = θ component of vorticity, s^{-1}, $\Omega^*_\theta/(\nu^*/r^{*2}_P)$
Ω^*_z = z^* component of vorticity, s^{-1}, $\Omega^*_z/(\nu^*/r^{*2}_P)$

Subscripts

∞ = freestream
p = plate

Superscript

$*$ = dimensional quantity

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*Department of Mechanical and Manufacturing Engineering; schaldar@rediffmail.com.

I. Introduction

HEAT transfer by free convection is a common phenomenon in many natural and engineering processes. The rate of free convection heat transfer from a surface depends on its geometry as well as its orientation. Correlations between Nu and Gr or Ra are available in almost all books on heat transfer for various combinations of surface geometry and its orientation. Gebhart et al. [1] discussed many free convection problems and correlations. Free convection about a flat plate with a width much larger than its height, commonly referred to as flat plate, in vertical orientation has possibly received the widest attention amongst free convection studies. Most of these studies deal with the analytical solution of boundary-layer equations based on the assumption that the boundary layer begins at the leading edge of the plate. Because boundary-layer equations are parabolic in nature, the solution is not influenced by activities downstream. Consequently, the results are applicable away from the leading and trailing edges of a plate of finite height. Martynenko et al. [2] provided a list of a large number of early studies on this topic. They also attempted to address the leading-edge issue with the help of a deformed longitudinal coordinate. Yang and Yao [3] introduced a double-deck structure at the trailing edge to account for its effect. Wright and Gebhart [4] addressed the leading-edge effects with the inclusion of the region below the plate into the solution domain. The equations were solved on a plane of transformed coordinates, and the obtained flowfield was explained with the help of a motion pressure gradient. However, the paper does not provide results on the Nusselt number. Vynnycky and Kimura [5] considered the thickness of the plate as well and solved the problem as a conjugate one. In a relatively recent paper, Andreozzi et al. [6] solved the elliptic form of the Navier–Stokes and energy equations by including into the numerical solution the regions upstream and downstream of the plate. It may be mentioned here that Andreozzi et al. considered a plate with an isoflux condition whereas most of the previous investigations dealt with an isothermal plate. All the studies cited here assume a plate with a width much larger than its height, rendering the problem two dimensional. A free convection about a circular plate oriented vertically has the additional difficulty of a three-dimensional nature. The author is not aware of any study on this topic, though it is one of the more commonly encountered geometries, such as the two ends of a cylindrical boiler drum or other storage vessel and the casing of electric motors. The present paper attempts to fill this gap in the literature.

II. Problem Description and Governing Equations

Figure 1 presents the schematic of the physical problem and the coordinate directions with the respective velocity components. The surface is flat, circular, and oriented vertically and is surrounded by an infinite medium of quiescent air. Because of the symmetry of the problem about the vertical plane passing through the plate center,