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# Technical Notes

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## Three-Dimensional Transient Radiative Transfer Modeling Using Discontinuous Spectral Element Method

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### Nomenclature

$A$ = area, m$^2$
$a$ = anisotropy parameter of linear anisotropic-scattering phase function
$c$ = speed of light, m/s
$G$ = integrated intensity, as defined by Eq. (8a), W/m$^2$
$\mathbf{H}$ = matrix, as defined in Eq. (7)
$h$ = one-dimensional standard nodal basis function
$\hbar$ = three-dimensional standard nodal basis function
$I$ = radiative intensity, W/(m$^2 \cdot$ sr)
$I_b$ = blackbody radiative intensity, W/(m$^2 \cdot$ sr)
$I_p$ = transient intensity on the boundary, W/(m$^2 \cdot$ sr)
$I_0$ = amplitude of transient intensity, W/(m$^2 \cdot$ sr)
$K$ = general hexahedral element
$K_{\mathrm{st}}$ = standard hexahedral element
$\mathbf{k}$ = unit direction vector of $z$ direction
$L_R$ = reference length scale, m
$\mathbf{M}$ = matrix, as defined in Eq. (7)
$M$ = number of discrete-ordinate directions
$N_{sk}$ = number of solution nodes on each element
$N_t$ = number of discretized time steps
$N_\theta$ = number of subdivisions for a zenith angle
$N_\varphi$ = number of subdivisions for an azimuthal angle
$\mathbf{n}_w$ = unit normal vector of the wall
$\mathbf{n}_{\partial K}$ = unit normal vector at the boundary of element $K$
$p$ = order of polynomial expansion
$q_z$ = heat flux of $z$ direction, as defined by Eq. (8b), W/m$^2$
$\mathbf{r}$ = vector of spatial coordinates ($x$, $y$, $z$)
$\tilde{S}$ = function, as defined in Eq. (7d), W/m$^3$
$T$ = temperature, K
$t$ = time, s
$t^*$ = dimensionless time $ct/L_R$, dimensionless time step
$u$ = unit step function
$V$ = volume, m$^3$
$w$ = weight of discrete-ordinates approximation, sr
$x$, $y$, $z$ = global coordinate system variables
$\mathbf{x}_{\mathrm{st}}$ = local coordinate vector ($x_{\mathrm{st}}$, $y_{\mathrm{st}}$, $z_{\mathrm{st}}$)
$x_{\mathrm{st}}$, $y_{\mathrm{st}}$, $z_{\mathrm{st}}$ = reference coordinate system variables
$\beta$ = extinction coefficient ($\kappa_a + \kappa_s$), m$^{-1}$
$\tilde{\beta}$ = function, as defined in Eq. (7c), m$^{-1}$
$\Delta t^*$ = dimensionless time step
$\theta$ = zenith angle
$\kappa_a$ = absorption coefficient, scattering coefficient, m$^{-1}$
$\kappa_s$ = scattering coefficient, m$^{-1}$
$\rho$ = bidirectional reflection function
$\sigma$ = Stefan–Boltzmann constant, W/(m$^2 \cdot$K$^4$)
$\tau_L$ = optical thickness, $\beta L$
$\tau_p$ = transmissivity
$\Phi$ = scattering phase function
$\phi$ = global nodal basis function
$\varphi$ = azimuthal angle
$\psi$ = map function, as defined by Eq. (5)
$\Omega$ = solid angle, sr
$\boldsymbol{\Omega}$ = unit vector of the radiation direction
$\omega$ = single scattering albedo

*Subscripts*

$i$ = mapped one-dimensional index
$i'$, $j'$, $k'$ = elemental spatial node index
$l$ = node index of the standard hexahedral element
$n$ = time step index
$w$ = value at wall

*Superscripts*

$m$, $m'$ = index of discrete-ordinate direction

## I. Introduction

TRANSIENT radiative transfer within a participating medium has attracted the interest of many researchers due to the availability of short-pulse lasers and their application to many emerging new technologies [1–3]. A number of methods have been developed to solve the transient radiative transfer equation (TRTE); such as the Monte Carlo method [4], the integral equation method [5], the discrete-ordinates method (DOM) [6,7], and the finite volume method (FVM) [8]. Among them, the methods based on the differential form of the TRTE, such as the DOM and the FVM, are efficient and easy to apply to problems with complex media and boundary conditions. However, the DOM and the FVM suffer from large false scattering, and the transient wave front cannot be captured efficiently and accurately.

Recently, based on a discontinuous Galerkin (DG) approach, Liu and Hsu [9] developed and analyzed transient radiative transfer in two-dimensional graded index media using a discontinuous finite element method (DFEM). In the DG approach, the approximation space is composed of discontinuous functions, which is expected to be ideal in solving transient radiative transfer problems and accurately capturing the sharp wave fronts. The DFEM showed good performance in solving the transient radiative transfer problems. As an advanced version of the DFEM, a discontinuous spectral element

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