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Cylindrical Geometry Verification Problem for Enclosure Radiation

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The development of a manufactured solution for enclosure radiation in an infinitely long circular cylinder with a nonparticipating medium is presented. This solution is then used to verify the correct implementation of the commonly used discrete enclosure equations. The circular cross section is approximated by a faceted geometry; the numbers of facets used are 4, 8, 16, 32, 64, and 128. The crossed-string method, which is exact in this application, is used to compute the view factors. Computational results using six levels of grid refinement suggest that the error norm between the integral equation solution and the discrete equation solution behaves as h^2 where h is a characteristic mesh size.

Nomenclature

A, A' = surface area, m^2, also region A

B, B_1, B_2 = region identifiers

a, b = integration limits

C, C_1, C_2 = constants

E_b = blackbody emissive power, W/m^2

E_{b_o} = blackbody emissive power evaluate at T_o, W/m^2; see Eq. (15)

e = error in numerical solution

F = view factor, dimensionless

F_{ij} = view factor between surfaces i and j, dimensionless

$f(x)$ = function of x

H = irradiation, W/m^2

H' = dimensionless irradiation; see Eq. (25)

h = characteristic mesh size, m

i = position index

J = radiosity, W/m^2

J' = dimensionless radiosity; see Eq. (24)

K = kernel, 1/m^2; see Eq. (6)

k = integer

$L_2(e)$ = $(\sum_{i=1}^{N} |e_i|^2)^{\frac{1}{2}}$

N = number of surface facets

p = order of grid convergence

q = heat flux, W/m^2

q' = dimensionless heat flux, see Eq. (26)

r = radius of cylinder, m

$\mathbf{r}_i, \mathbf{r}_j$ = position vector on surface i or j

S = distance between surface elements, m

s = arc length, m

T = temperature, K

T_i = temperature of facet i, K

T_o = enclosure temperature at $\theta = 0$, K

T' = dimensionless temperature; see Eq. (27)

x_0, x_1 = integration limits; see Eq. (30)

α = angle, deg

β = angle, deg

ϵ = emittance

θ, θ' = angle, deg

ξ = a value in the interval $a < \xi < b$; see Eqs. (30) and (31)

ρ = reflectance

σ = Stefan–Boltzmann constant, W/m$^2 \cdot$ K^4

I. Introduction

MODERN computational software may contain tens to hundreds of thousands of lines of code. To develop confidence that there are no coding mistakes, a formal code verification process should be an integral part of the code development process. For the purposes of this paper, code verification is the *process of ensuring that the model equations are implemented and solved correctly.* The ideal time for code verification is while the software is being developed. All code development teams do some amount of code verification during the development process. Unfortunately, the end user of the software may not have access to this information. If, for example, one models mission critical components that may get too hot or have stress levels that are too high, then the end user should do some code verification.

Significant strides have been made in the code verification arena. Books with "verification" in the title are starting to appear (Roache [1] and Knupp and Salari [2]) as the technology matures. The most commonly used approach for code verification involving ordinary and partial differential equations as well as integral equations is the *order verification procedure*. If h is a characteristic mesh size, the discretization error e in the numerical solution behaves as h^p, where p is the order of convergence. Mathematically, this can be expressed as

$$e = Ch^p + \text{HOT} \tag{1}$$

where C is a constant, and HOT refers to higher-order terms. A sequence of meshes is used to estimate p from the errors in the numerical results. If the computed p does not match the theoretical p, then there can be errors other than discretization errors or there

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