JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 23, No. 4, October–December 2009

Numerical Simulation of Developing Compressible Turbulent Flow with Heat Transfer

A. Nouri-Borujerdi* and M. Ziaei-Rad†
Sharif University of Technology, 11365-9567 Tehran, Iran
and
J. R. Seume‡
Leibniz University Hannover, 30167 Hannover, Germany

DOI: 10.2514/1.41669

This study investigates the effects of wall heating and skin friction on the characteristics of a compressible turbulent flow in developing and developed regions of a pipe. The numerical solution is performed by finite-element-based finite volume method applied on unstructured grids. A modified κ-ε model with a two-layer equation for the near-wall region and a compressibility correction are used to predict turbulent viscosity. The results show that shear stress in fully developed flow is nearly constant from the centerline up to 75% of the pipe radius, then increases sharply next to the wall, and the ratio of the turbulent viscosity to the molecular one is less than 0.2. Under a uniform wall heat flux condition, the friction factor decreases in the entrance region and will be fully developed after $Z/D > 50$, but the Nusselt number increases first and then will be fully developed after $Z/D > 10$. In addition, the heat flux accelerates the developing compressible flow and causes the entrance length to decrease, unlike the incompressible flow.

Nomenclature

A	=	hexagonal area, m^2
A	=	Jacobian matrix
b	=	energy flux by force and heat, W/m^2
C	=	triangle area, m^2
c	=	speed of sound, m/s
C	=	constant
D	=	pipe diameter, m
D	=	destruction, 1/s
e	=	internal energy, J/kg
F	=	inviscid vector flux
f	=	friction factor
H	=	total enthalpy, J/kg
H	=	Heaviside step function
k	=	thermal conductivity, W/m · K
L	=	pipe length, m
l	=	length scale, m
M	=	Mach number ($u/\sqrt{\gamma RT}$)
N	=	viscous flux vector, number of grid nodes
Nu	=	Nusselt number (hD/k)
n	=	unit normal vector
P	=	production
p	=	pressure, Pa
Pr	=	Prandtl number ($\mu C_p/k$)
Q	=	conservative variables vector
q''	=	heat flux rate, W/m^2
R	=	pipe radius, m
R	=	eigenvector matrix
Re	=	Reynolds number ($\rho \bar{u} D/\mu$)
r	=	radial direction
S	=	source vector, W/m^3
T	=	temperature, K
t	=	time, s
u	=	velocity in the z direction, m/s
V	=	velocity vector, m/s
y	=	distance measured from wall inward, m
z	=	axial direction
γ	=	specific heat ratio
Δ	=	difference
δ	=	boundary-layer thickness, m
ε	=	turbulent dissipation energy, W/kg
κ	=	turbulent kinetic energy, m^2/s^2
Λ	=	eigenvalues matrix,
μ	=	dynamic viscosity, kg/m · s
ρ	=	density, kg/m^3
τ	=	shear stress, Pa
ϕ	=	shape function
ψ	=	general function
Ω	=	computational domain

Subscripts

c	=	critical, centerline, cutoff
fd	=	fully developed
h	=	discretized computational domain
i, j, k	=	direction, counter
in	=	inlet
L	=	lower cell index
n	=	normal to boundary
out	=	outlet
R	=	upper cell index
r, θ	=	cylindrical coordinates
S	=	Sutherland constant, compressibility
t	=	turbulent, tangential
w	=	wall
0	=	reference
1,2,3	=	indices for triangle vertices

Superscripts

n	=	time-step iteration
ˆ	=	Roe-average quantity

Received 17 October 2008; revision received 12 May 2009; accepted for publication 22 May 2009.

*Professor, School of Mechanical Engineering, Azadi Avenue; anouri@sharif.edu.

†Ph.D. Student, School of Mechanical Engineering, Azadi Avenue; ziaeirad@mech.sharif.edu.

‡Professor, Institute of Turbomachinery and Fluid Dynamics, Appelstrasse 9; seume@tfd.uni-hannover.de.