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Two Methods for Solving Radiative Transfer in a Two-Dimensional Graded-Index Medium

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Two methods are presented for the study of radiative heat transfer in a two-dimensional graded-index semitransparent medium. One is a combination of the cell model and the ray-tracing technique, and the other is a combination of the linear refractive-index cell model and the discrete curved-ray-tracing technique. The medium is discretized into many cells. The constant-refractive-index assumption is used in the first model and the linear approximation of refractive index is used in the second model. We have compared the results and the calculation efficiency of these two methods, as well as the heat transfer at radiative equilibrium. The results of the two methods show little difference. We also studied the radiative heat transfer properties of the medium, and the results show that the refractive-index distribution has great influence on the temperature field. That is, the medium with various types of graded index has a higher or lower temperature than the medium with a constant refractive index. The medium also has a flat temperature field or a sharper temperature field than the constant-refractive-index medium. This is due to internal total reflection in the graded-index medium.

Nomenclature

a_{im}	=	coefficient denoting the influence of node m to node i
b	=	index of a boundary segment, 1–$(2N_x + 2N_y)$
c_{ik}	=	coefficient denoting the influence of boundary node k to node i
I	=	radiative intensity, $\mathrm{W/m^2 \cdot sr}$
i	=	index of a cell in the x direction
j	=	index of a cell in the y direction
L	=	side length of the medium, m
m	=	index of a cell of the medium, 1–N_xN_y
mi	=	ith cell of the medium through which the ray passes ($i = 1$–p and p varies for each ray)
N	=	number of nodes
$n_{i,j}$	=	refractive index in a cell
p	=	number of the cell through which the ray passes
q	=	qth segment of the boundary that is at the end of the ray-tracing
s	=	distance along the ray trajectory, m
s_i	=	distance a ray travels when passing through the ith cell ($i = 1 \sim p$ and p varies for each ray), m
T	=	temperature, K
$T_{\max}$	=	maximum value of the temperature in the medium, K
$T_{\min}$	=	minimum value of the temperature in the medium, K
T_{standard}	=	temperature calculated by the curved-ray-tracing method in a 40×40 spatial mesh and a 70×70 directional mesh, K
x, y	=	Cartesian coordinates, m
δ	=	relative error of the calculated temperature from T_{standard} or benchmark solution
$\Delta x, \Delta y$	=	length of a cell in the x direction and y direction, m
ΔT	=	temperature difference from T_{standard}, K
θ	=	polar angle, deg
θ_i	=	discrete polar angle ($i = 1$–N_θ), deg
κ	=	absorption coefficient, $\mathrm{m^{-1}}$
σ	=	Stefan–Boltzmann constant, 5.67×10^{-8} $\mathrm{W/(m^2 \cdot K^4)}$
φ	=	azimuthal angle, deg
φ_j	=	discrete azimuthal angle ($j = 1$–N_φ), deg
Ω	=	local direction vector

Subscripts

b	=	black body
bi	=	segment of boundary, $i = 1$–$(2N_x + 2N_y)$
bq	=	qth segment of the boundary which is at the end of the ray-tracing
m	=	medium
mi	=	ith cell of the medium which the ray passes through ($i = 1$–p and p varies for each ray)
x, y	=	x and y directions
θ	=	polar angle
φ	=	azimuthal angle

I. Introduction

RADIATIVE heat transfer in a graded-index (GRIN) medium can be found in many processes, such as the heating of glass and thermal protective coatings, manufacturing of waveguide materials, and ray propagation through the atmosphere, as well as the optical measurement in flames and other semitransparent media. The rays propagating inside a GRIN medium are not straight lines, but curved lines, which are governed by minimization of the Fermat principle [1]. Based on this principle, total reflection will occur within the medium. The refractive index has significant influence on the radiative transfer in the medium.

Recently, some researchers have turned their efforts toward radiative transfer in a GRIN medium. In 2000, Ben Abdallah and Le Dez [2] put forward a curved-ray-tracing technique to solve for the radiation transfer in a GRIN medium and first analyzed the thermal emission of a semitransparent slab with a variable spatial refractive index. By this method, they also investigated radiation heat transfer and coupled radiation–conduction inside a semitransparent slab with a variable spatial refractive index [3,4] as well as the thermal emission of a two-dimensional rectangular cavity [5]. In 2001, Huang et al. [6] proposed a combination of a pseudo-source-adding method and a ray-splitting/tracing method to study the apparent thermal emission of a semitransparent slab with a graded index and gray diffuse substrate. They also studied the temperature field of a

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