40th Thermophysics Conference
23 - 26 June 2008, Seattle, Washington

AIAA 2008-4256

2008-4256

High-Order Discontinuous Galerkin Method for Boltzmann Model Equations

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High-order Runge-Kutta discontinuous Galerkin (DG) method is applied to the kinetic model equations describing rarefied gas flows. A conservative DG discretization of non-linear collision relaxation term is formulated for Bhatnagar-Gross-Krook and ellipsoidal statistical models. The numerical solutions using RKDG method of order up to four are obtained for two flow problems: the heat transfer between parallel plates and the normal shock wave. The convergence of RKDG method is compared with the conventional second-order finite volume method for the heat transfer problem. The normal shock wave solutions obtained using RKDG are compared with the experimental measurements of density and velocity distribution function inside the shock.

I. Introduction

Many practical applications in non-equilibrium gas dynamics require the kinetic description of gas flow phenomena that are governed by the Boltzmann kinetic equation. For applications involving unsteady rarefied gas flows, the direct simulation Monte Carlo method for stochastic solution of the Boltzmann equation may not be practical and the deterministic numerical solution of the Boltzmann-BGK equation can be more useful. However, the computational time and memory required for the deterministic kinetic modeling are large due to the multi-dimensionality of the Boltzmann-BGK equation whose phase space involves both velocity and spatial coordinates. Therefore, the development of high-order space/time discretization methods for the Boltzmann-BGK equation is very important.

Currently, there are two main approaches to obtaining high-order space/time discretization for solution of partial-differential equations. The first approach is based on weighted essentially non-oscillatory (WENO) finite volume and finite different methods. The WENO approach was first suggested in mid-1990s by Jiang and Shu[1] and is currently the most widely used high-order spatial reconstruction technique for Euler and Navier-Stokes equations.[2–4] The main idea of the WENO approach consists in applying an adaptive stencil for high-order reconstruction of values at cell boundaries using volume-averaged values. The stencil adaptation in WENO is based on a convex combination of all possible stencils with weights based on the local smoothness of the numerical solution. Recently, high-order WENO schemes for finite difference method has been applied for the solution of the Boltzmann-Poisson equation for simple rectangular geometries.[5]

Another class of high-order numerical methods is the Runge-Kutta discontinuous Galerkin (RKDG) finite element method. The RKDG method has been recently shown in Ref.[6] to have a significant advantage in computational efficiency over the WENO approach for a number of hyperbolic conservation laws but has not yet been applied for the Boltzmann equation. The discontinuous Galerkin method is the finite element method with a test space of piecewise-continuous functions that allows for discontinuities to exist at element boundaries. In the case when polynomials are used as the test functions, the order of the space discretization of the DG method is determined by the highest degree of the polynomial basis and can be implemented for

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